

ECE101 F19 Lecture 4 Oct 8, 2019

Review of what we have covered up to now:

$$\text{Ohm's Law } dV = R di \quad i = RI$$

$$dI = \frac{1}{R} dV = g dV \quad i = gV$$

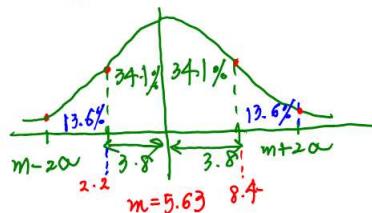
KCL at Node: $\sum_{k=1}^N i_k = 0$



$Q = 1$ (98 students)

$$\text{Avg} = 5.63$$

$$\sigma = 3.80$$



Power P [W]

$$\begin{aligned} &+ \downarrow i_K \\ \text{Circuit: } &V_K - \\ &P_K = V_K \cdot I_K \\ &\equiv (R_K I_K) \cdot I_K = R I_K^2 \\ &V_K = R_K I_K \\ &\equiv V_K \left(\frac{1}{R} V_K \right) = \frac{V_K^2}{R_K} \\ &I_K = \frac{1}{R} V_K \end{aligned}$$

Energy $W(t)$ (or $E(t)$)

$$W(t) = \int_0^t P(\tau) d\tau$$

Linear resistor

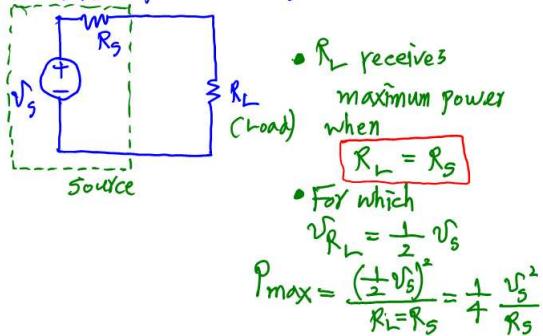
$$\begin{aligned} &V \\ &i \\ &\text{Straight Line Passing thru origin} \\ &V = R i \quad (\text{Ohm's law}) \end{aligned}$$

$$\begin{aligned} &i = \frac{1}{R} V = g V \\ &R[\Omega], g[S] \end{aligned}$$

Nonlinear resistor

$$\begin{aligned} &i \\ &V \\ &i = I_o (e^{KV} - 1) \\ &\text{diode including LED} \end{aligned}$$

Maximum Power Transfer Theorem

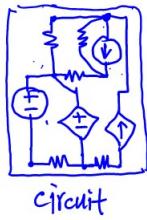


Circuit elements

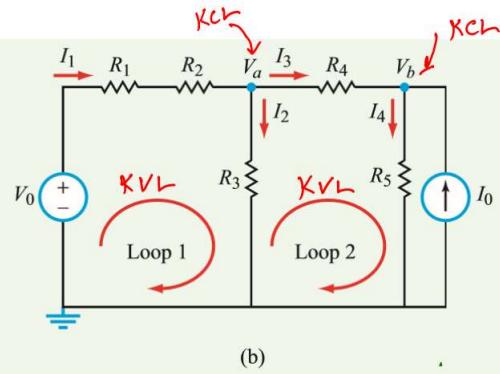
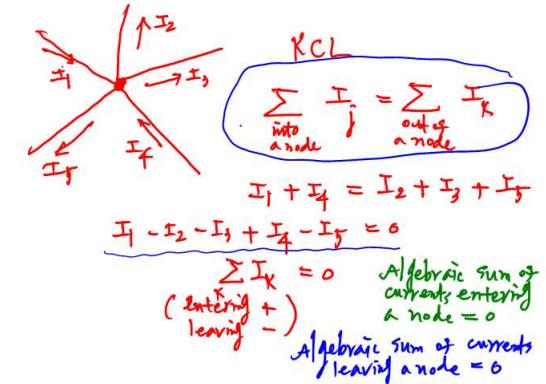
	Linear
	DC
	DC
	CCVS
	CCCS

Circuit Analysis : (Circuit formed by connection of elements)

- * To find currents through elements (e.g., R, voltage source) or voltages across elements (e.g., R, current sources)



- Power consumed or generated
- Energy consumed or delivered



(b) KVL equations

The circuit contains two independent loops that do not contain the current source I_0 . The associated KVL equations are:

$$\begin{aligned} -V_0 + I_1 R_1 + I_1 R_2 + R_3 I_2 &= 0 \quad (\text{Loop 1}), \\ -I_2 R_3 + I_3 R_4 + I_4 R_5 &= 0 \quad (\text{Loop 2}). \end{aligned}$$

4 unknowns
2 equations

Alternatively, we can replace either of the two loop equations with the KVL equation for the perimeter loop that includes both of them, namely the loop that starts at the ground node, then goes clockwise through V_0 , R_1 , R_2 , R_4 , and R_5 , and back to the ground node. Either approach leads to the same final result.

(c) KCL equations

We have two extraordinary nodes (in addition to the ground node). We designate their voltages as shown in Fig. 2-14(b). With current defined as positive when entering a node, their KCL equations are

$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \quad (\text{Node } a), \\ I_3 - I_4 + I_0 &= 0 \quad (\text{Node } b). \end{aligned}$$

additional
2 equations

(d) Arrange equations in matrix form

4 equations

$$\Rightarrow \begin{bmatrix} (R_1 + R_2) & R_3 & 0 & 0 \\ 0 & -R_3 & R_4 & R_5 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \\ 0 \\ -I_0 \end{bmatrix}$$

4 UNKNOWNs

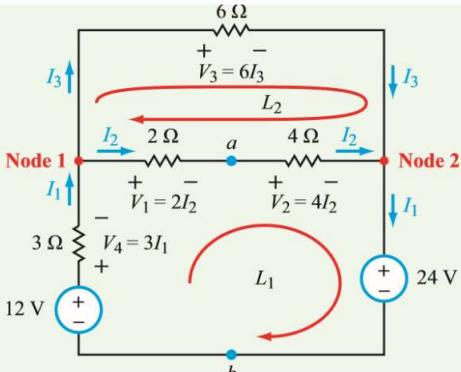
This is in the form

$$AI = B.$$

(e) Matrix inversion $I = A^{-1} B = \frac{adj A}{|A|} B$

After replacing the sources and resistors with their specified numerical values, matrix reduction, per MATLAB, MathScript, or the procedure outlined in Appendix B-2, leads to

$$\begin{aligned} I_1 &= 1.1 \text{ A}, & I_2 &= 0.9 \text{ A}, \\ I_3 &= 0.2 \text{ A}, & I_4 &= 1 \text{ A}. \end{aligned}$$



(b) After assigning currents at nodes 1 and 2

In terms of the labeled voltages, application of KVL around the two loops gives

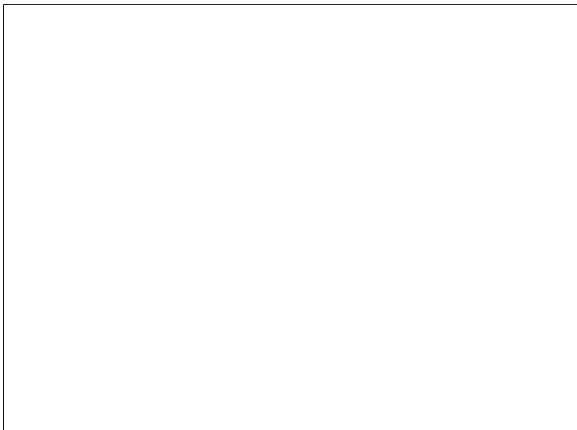
$$-12 + V_4 + V_1 + V_2 + 24 = 0, \quad (\text{KVL for Loop 1}) \quad (2.14a)$$

$$V_3 - V_2 - V_1 = 0. \quad (\text{KVL for Loop 2}) \quad (2.14b)$$

Using Ohm's law for the four resistors, the two KVL equations become

$$-12 + 3I_1 + 2I_2 + 4I_2 + 24 = 0, \quad (\text{KVL for Loop 1}) \quad (2.15a)$$

$$6I_3 - 4I_2 - 2I_2 = 0. \quad (\text{KVL for Loop 2}) \quad (2.15b)$$



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$$6I_3 - 4I_2 - 2I_2 = 0. \quad (\text{KVL for Loop 2}) \quad (2.15b)$$

**3 unknowns
2 equations
additional eq.**

The two simultaneous equations contain three unknowns, namely I_1 to I_3 . A third equation is supplied by KCL at node 1 or node 2:

$$I_1 = I_2 + I_3. \quad (\text{KCL @ node 1 or 2}) \quad (2.16)$$

Equations (2.15a), (2.15b), and (2.16) constitute 3 equations in 3 unknowns. We can solve for I_1 to I_3 either by the substitution method or by matrix inversion (Appendix B). To apply the latter, we need to cast the three equations in standard form:

$$\begin{cases} 3I_1 + 6I_2 &= -12, \\ -6I_2 + 6I_3 &= 0, \\ I_1 - I_2 - I_3 &= 0. \end{cases}$$

**3 unknowns
3 equations**

$$\begin{aligned} 3I_1 + 6I_2 &= -12, \\ -6I_2 + 6I_3 &= 0, \\ I_1 - I_2 - I_3 &= 0. \end{aligned}$$

In matrix form:

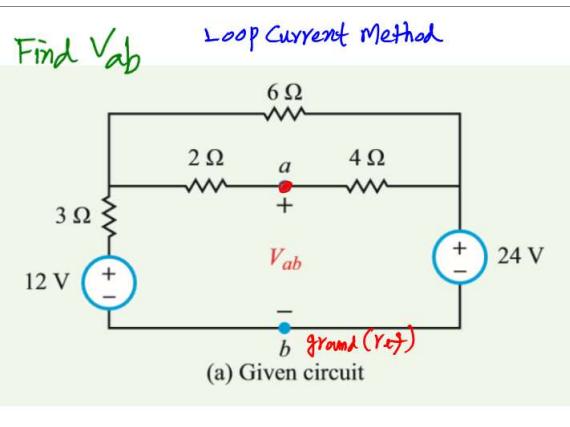
$$\begin{bmatrix} 3 & 6 & 0 \\ 0 & -6 & 6 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 0 \end{bmatrix}.$$

Matrix inversion, as outlined in Appendix B, leads to

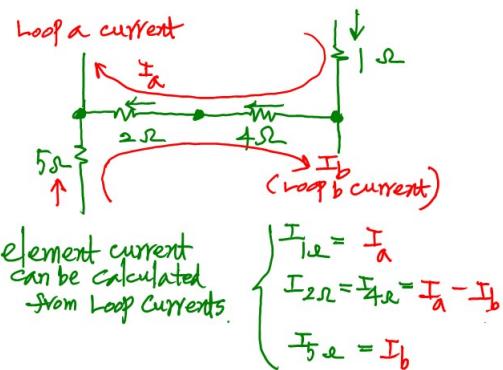
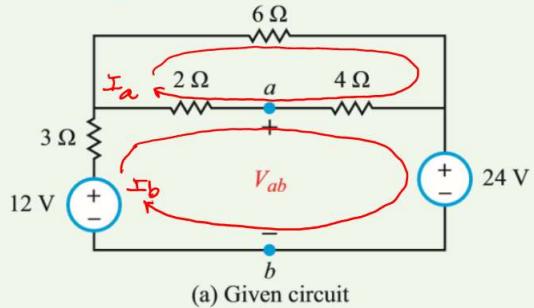
$$I_1 = -2 \text{ A}, \quad I_2 = -1 \text{ A}, \quad I_3 = -1 \text{ A}.$$

$V_2 = 12V - 6I_2 = 12 - 6(-1) = 18 \text{ V}$

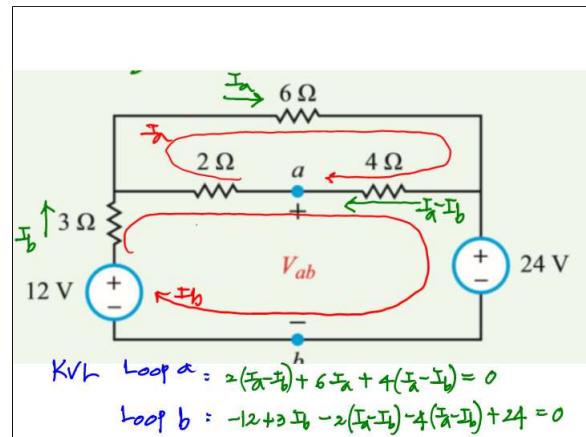
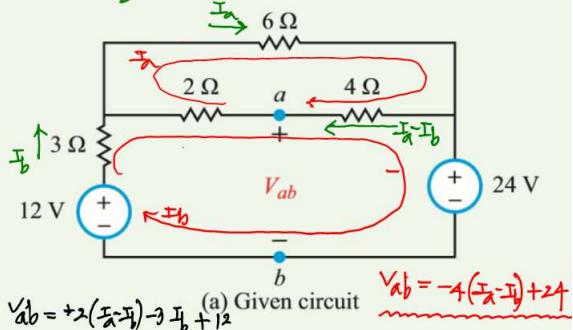
$V_{ab} = 24V - 6I_3 = 24 - 6(-1) = 30 \text{ V}$



Find V_{ab}



Find V_{ab}



$$12I_a - 6I_b = 0$$

$$-6I_a + 9I_b = -12$$

$$\Rightarrow 2I_a - I_b = 0 \quad (\Rightarrow I_b = 2I_a)$$

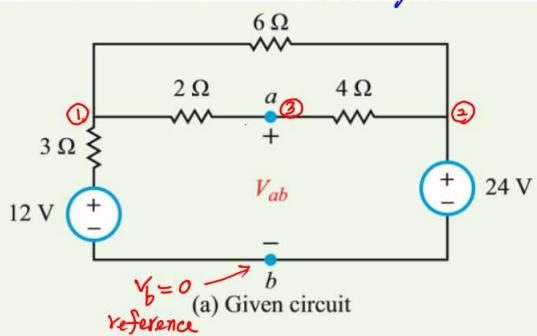
$$-2I_a + 3I_b = -4$$

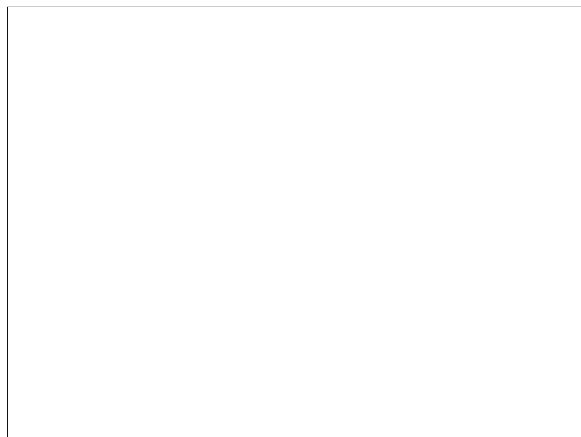
$$\text{and } 2I_a = I_b \Rightarrow I_a = \frac{I_b}{2} = -1 \text{ A}$$

$$V_{ab} = -4(I_a - I_b) + 24$$

$$= -4(-1 - (-2)) + 24 = 20 \text{ V}$$

Alternative Method: Nodal Analysis





Matrix Equation

$$\tilde{A} \tilde{x} = \tilde{b}$$

$$\tilde{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}, \quad \tilde{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}$$

$$x = \frac{\tilde{x}}{\tilde{A}^{-1} \tilde{b}}$$

$$= \frac{\text{Adj } \tilde{A}}{|\tilde{A}|} \tilde{b}$$

Cramer's rule

$$x_k = \frac{\begin{vmatrix} a_{11} & \dots & a_{1k} & \dots & a_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nk} & \dots & a_{nN} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nN} \end{vmatrix}}$$

(Note: The diagram shows the k-th column of A underlined in green.)

KCL node ②: $\frac{V_0 - V_1}{2} - \frac{V_0 - 24}{6} = 0$ (1)

node ①: $\frac{12 - V_1}{3} - \frac{12 - 24}{6} - \frac{V_1 - 24}{2} = 0$ (2)

(1) $\Rightarrow 2V_0 - 2V_1 - V_0 + 24 = 0$
 $2V_0 - 3V_1 = -24$

(2) $\Rightarrow 2(12 - V_1) - (V_0 - 24) - 3(V_0 - 24) = 0$
 $24 - 2V_0 - V_0 + 24 - 3V_0 + 3V_0 = 0$
 $-6V_0 + 2V_0 = -48$
 $-4V_0 = -48$
 $V_0 = 12$

$A \tilde{x} = \tilde{b}$
 $\tilde{A}^{-1} \tilde{A} \tilde{x} = \tilde{A}^{-1} \tilde{b}$
 $\tilde{A}^{-1} = \text{Adj } \tilde{A} / |\tilde{A}|$
 $\begin{bmatrix} 2 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \end{bmatrix} = \begin{bmatrix} -24 \\ -16 \end{bmatrix}$
 $\begin{bmatrix} V_0 \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}^{-1} \begin{bmatrix} -24 \\ -16 \end{bmatrix} = \begin{bmatrix} 18 \\ 20 \end{bmatrix}$

Cramer's Rule

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$\begin{bmatrix} 2 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \end{bmatrix} = \begin{bmatrix} -24 \\ -16 \end{bmatrix}$$

$$V_0 = \frac{\begin{vmatrix} 2 & -24 \\ -2 & -16 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ -2 & 1 \end{vmatrix}} = \frac{2 \times (-16) - (-24) \times (-2)}{2 \times (-3) - (-2) \times (-2)} = \frac{-32 - 48}{-4} = \frac{-80}{-4} = 20 = V_{ab} \text{ (Ans)}$$

Equivalent Circuits chap 3 (70-88)

(a) Circuit diagram showing three nodes (1, 2, 3) connected to resistors R_1 , R_2 , R_3 , R_L , and voltage sources v_1 , v_2 , v_3 . Node 1 is at the bottom, Node 2 is in the middle, and Node 3 is at the top.

(b) Simplified equivalent circuit diagram showing Node 1 and Node 2. The voltage across the equivalent source is $v_{eq} = v_1 - v_2 + v_3$ and the equivalent resistance is $R_{eq} = R_1 + R_2$.

(a) Original circuit

(b)

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

$$i_2 = \left(\frac{R_{eq}}{R_2} \right) i_s$$

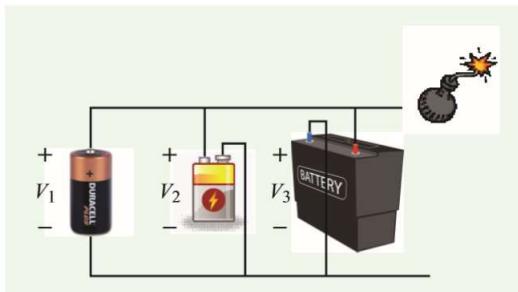


Figure 2-24: This is an unrealizable circuit unless all voltage sources have identical voltages and polarities; that is, $V_1 = V_2 = V_3$.

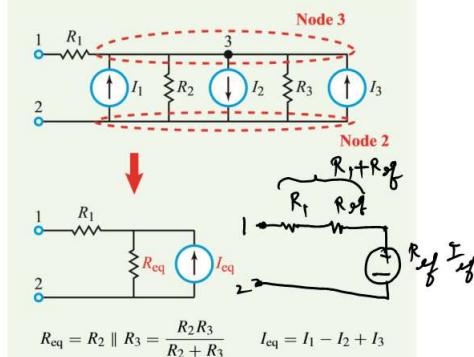
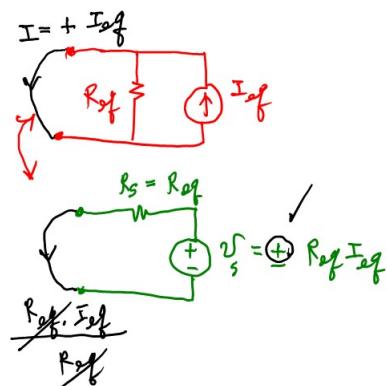


Figure 2-25: Adding current sources connected in parallel.



$$G_{\text{eq}} = \frac{1}{R_{\text{eq}}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = G_1 + G_2 + G_3.$$

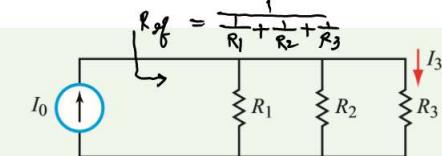


Figure 2-26: Circuit of Example 2-10.

$$I_0 = ? \quad \frac{I}{\sigma} = \frac{q_3}{q_1 + q_2 + q_3}$$

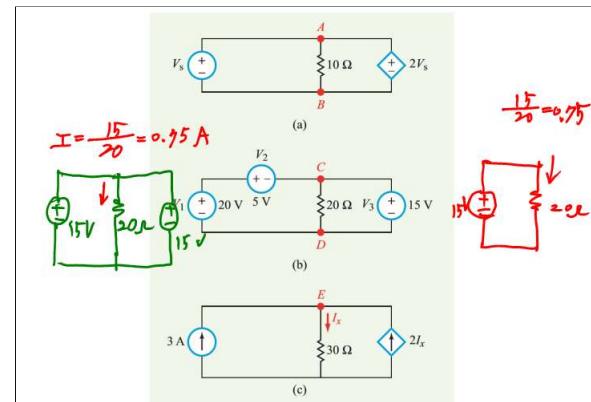
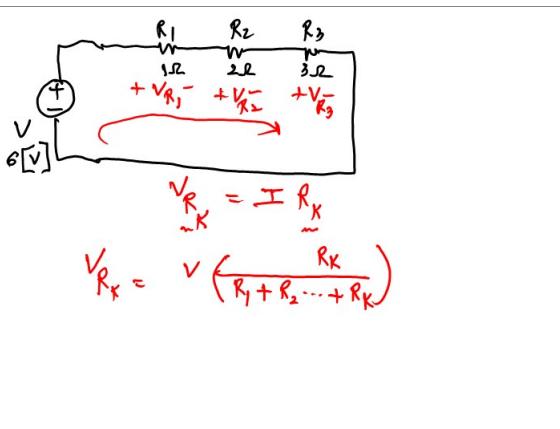
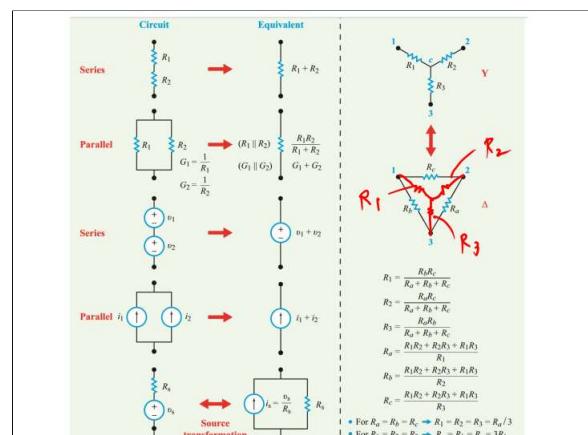
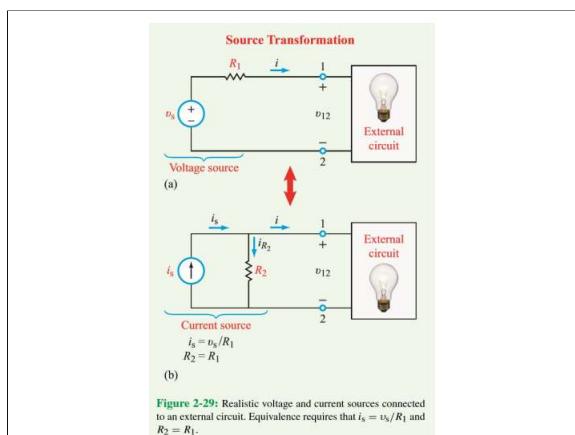
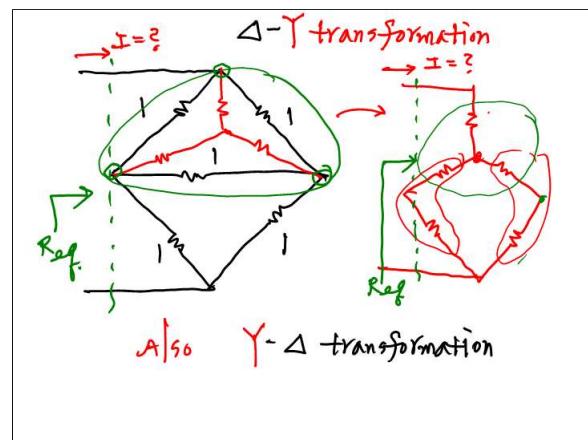
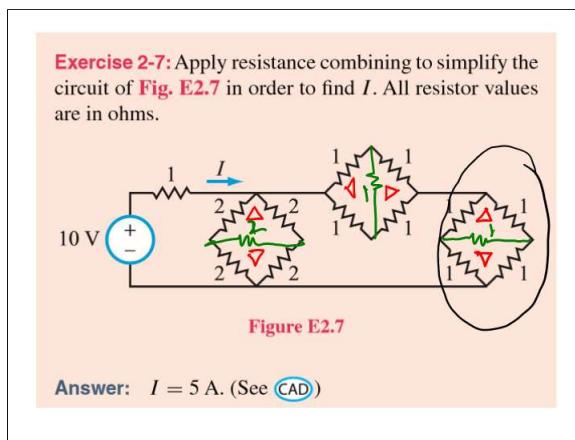
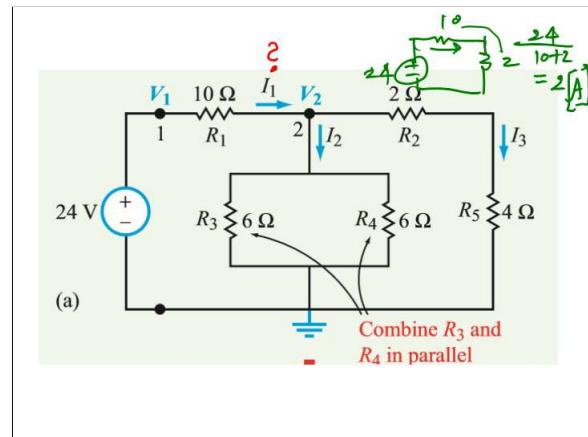
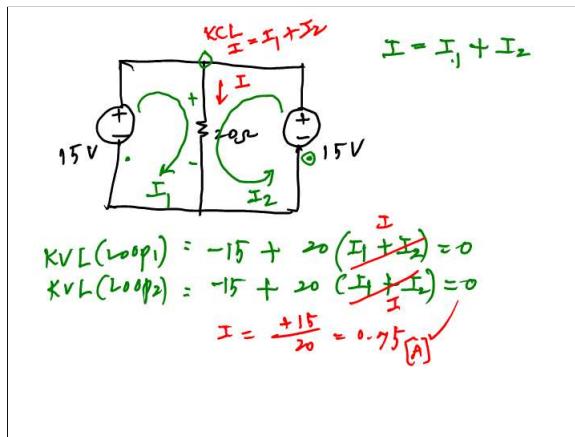
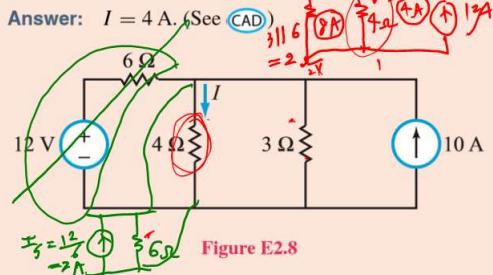


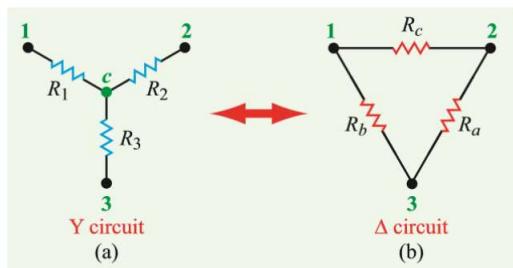
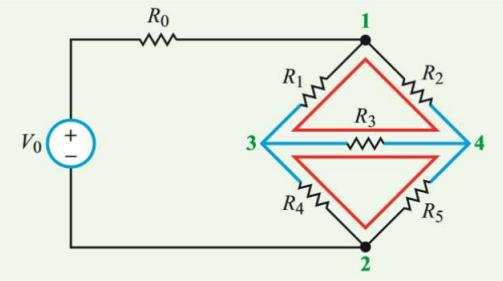
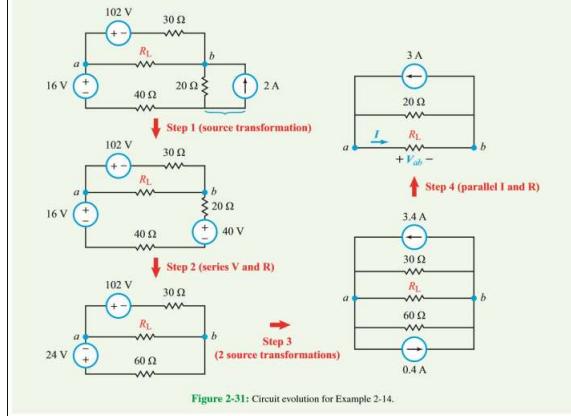
Figure 2-27: Circuits of Example 2-11.



Exercise 2-8: Apply source transformation to the circuit in Fig. E2.8 to find I .

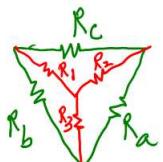


Answer: $I = 4 \text{ A.}$ (See CAD)



2-4.1 $\Delta \rightarrow Y$ Transformation

Solution of the preceding set of equations provides the following expressions for R_1 , R_2 , and R_3 :



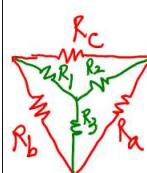
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad (2.42a)$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \quad (2.42b)$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \quad (2.42c)$$

2-4.2 $Y \rightarrow \Delta$ Transformation

When applied in the reverse direction, from Y to Δ , the associated transformation relations are given by the following expressions.



$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1} \quad (2.43a)$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2} \quad (2.43b)$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3} \quad (2.43c)$$

Find I

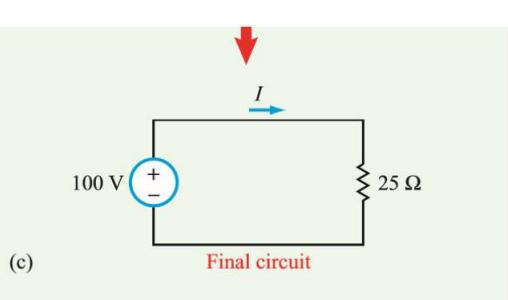
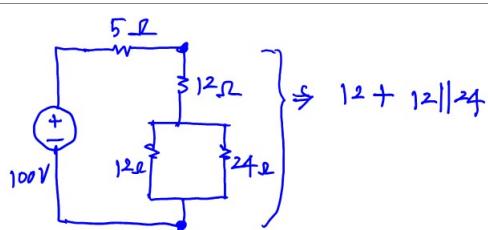
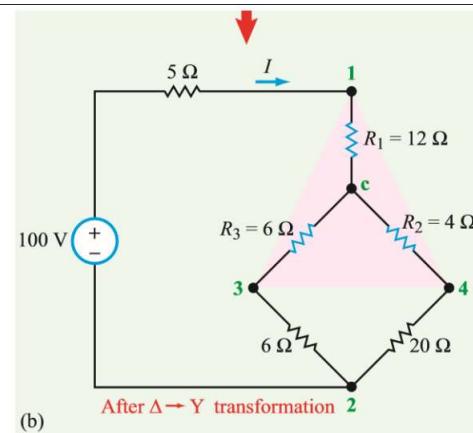
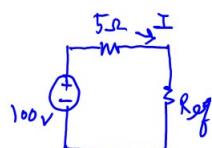
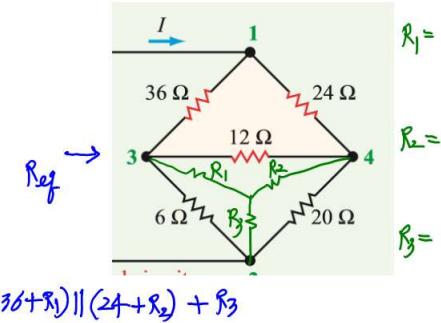
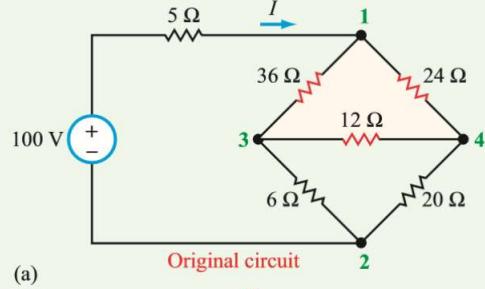


Figure 2-35: Example 2-15 circuit evolution.