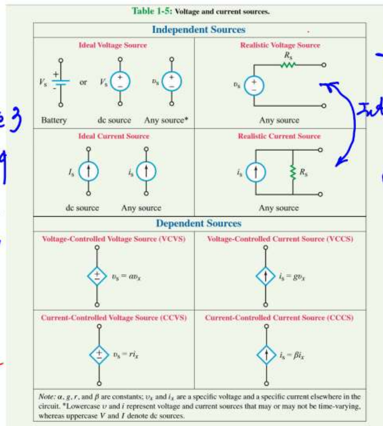
ECE101 F19

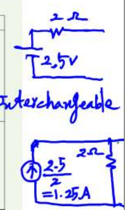
Lecture 3

Oct 3, 2019

Quiz 1 Today

HW#2 watch webpage this evening





HW#2 Assigned on Oct 3.

1) Prob. 2.3
2) Prob 2.7
3) Prob 2.11
4) Prob 2.17
5) Prob 2.19
6) Prob 2.23
7) Prob 2.25
8) Prob 2.29
9) Prob 2.34
10) Prob. 2.38

one of these will be in QZ 2 on Oct. 10

KVL $+6 +12 +6 -24 = 0$ ✓

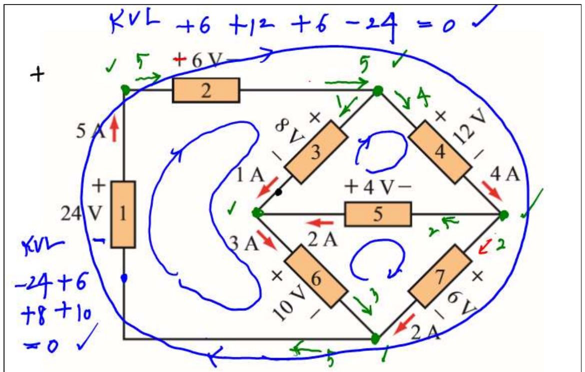
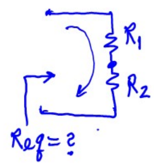
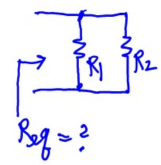


Figure P1.28: Circuit for Problem 1.28.

KVL $-24+6+8+10 = 0$ ✓

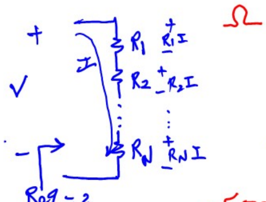


$R_1 + R_2 = R_{eq}$



$R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$

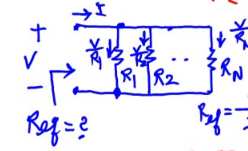
Ω



$R_{eq} = R_1 + R_2 + R_3 \cdots + R_N = \sum_{k=1}^{N} R_k$

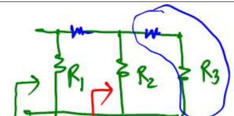
$V = R_1 I + R_2 I + \cdots + R_N I = \underbrace{(R_1 + R_2 + \cdots + R_N)}_{R_{eq}} I$

G [℧ mho, S siemen]



$I = \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_N} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right) V$

$R_{eq} = \frac{V}{I} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)}$



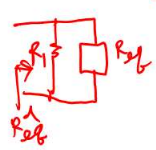
$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$\Rightarrow R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

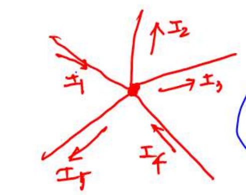
$G = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_{eq}}$

$= \frac{1}{\frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 R_2 R_3}}$

$R_{eq} = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = \square \Leftarrow \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$



$\hat{R}_{eq} = R_1 \parallel R_{eq} = \frac{R_1 R_{eq}}{R_1 + R_{eq}}$



KCL

$$\sum_{\text{into a node}} I_j = \sum_{\text{out of a node}} I_k$$

$$I_1 + I_4 = I_2 + I_3 + I_5$$

$$I_1 - I_2 - I_3 + I_4 - I_5 = 0$$

$$\sum_k I_k = 0$$

(entering +, leaving −)

Algebraic sum of currents entering a node = 0

Algebraic sum of currents leaving a node = 0

Write the KCL equations at nodes 1 through 5 in the circuit of **Fig. 2-10**.

Solution:

KCL / KVL

At node 1: $-I_1 - I_3 + I_5 = 0$
At node 2: $I_1 - I_2 + 2 = 0$
At node 3: $-2 - I_4 + I_6 = 0$
At node 4: $-5 - I_5 - I_6 = 0$
At node 5: $I_3 + I_4 + I_2 + 5 = 0$

5 eq, 6 unknown

$$-R_3 I_3 - V_1 + R_1 I_1 + R_2 I_2 = 0$$

6th eq.

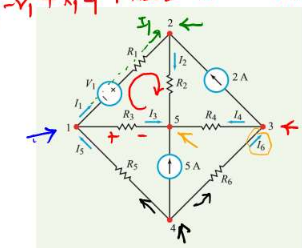
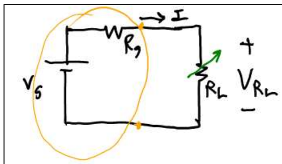


Figure 2-10: Circuit for Example 2-3.

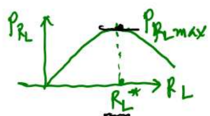


$$P_L = V_{R_L} \cdot I = (R_L \cdot I) \cdot I = I^2 R_L = \left(\frac{V_{R_L}}{R_L}\right)^2 R_L = \frac{V_{R_L}^2}{R_L}$$

$$V = R_s I + R_L I$$

$$\Rightarrow I = \frac{V_s}{R_s + R_L}$$

$$P_L = I^2 R_L = \left(\frac{V_s}{R_s + R_L}\right)^2 \cdot R_L = V_s^2 \cdot \frac{R_L}{(R_s + R_L)^2}$$



$$\frac{\partial P_{R_L}}{\partial R_L} = 0 \Rightarrow R_L^*$$

$$\boxed{R_L^* = R_s}$$

$$P_L = I^2 R_L = \left(\frac{V_s}{R_s + R_L}\right)^2 R_L = V_s^2 \frac{R_L}{(R_s + R_L)^2}$$

$$\frac{\partial P_L}{\partial R_L} = V_s^2 \left(\frac{(R_s + R_L)^2 - R_L \cdot 2(R_s + R_L)}{(R_s + R_L)^4} \right)$$

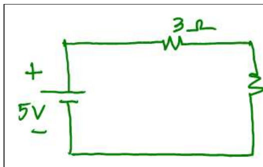
$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

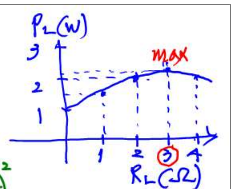
$$= V_s^2 \frac{(R_s + R_L)(R_s + R_L - 2R_L)}{(R_s + R_L)^4}$$

At P_L^{max}, $\frac{\partial P_L}{\partial R_L} = 0$

$$\Rightarrow (R_s - R_L) = 0$$

$$\Rightarrow \boxed{R_L = R_s} = R_L^*$$





R_L	$P_{R_L} = R_L\left(\frac{5}{3+R_L}\right)^2$
1	$1 \cdot \left(\frac{5}{3+1}\right)^2 = 1.5625$ W
2	$2\left(\frac{5}{3+2}\right)^2 = 2.0$ W
3	$3\left(\frac{5}{3+3}\right)^2 = 2.083$ W
4	$4\left(\frac{5}{3+4}\right)^2 = 2.041$ W

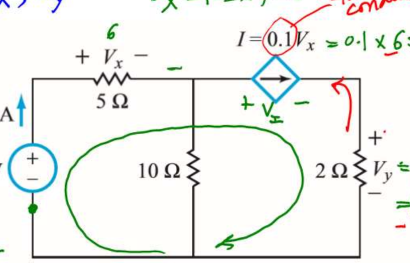
max $R_L = 3 = R_s$

Find V_x, V_y

$V_x = 1.2 \times 5 = 6$ V

transfer conductance

$I = 0.1 V_x = 0.1 \times 6 = 0.6$ A

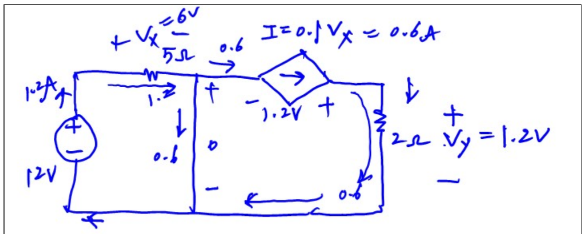


$V_y = 2(0.6) = 1.2$ V

KVL
$-12 + 6 + V_I + 1.2 = 0$
$V_I = 12 - 7.2 = 4.8$ V

Figure P1.38: Circuit for Problem 1.38.

Transistor = transfer resistor

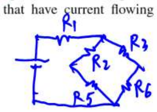


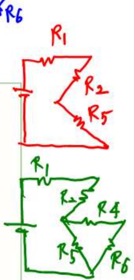
1.42 For the circuit in **Fig. P1.42**, generate circuit diagrams that include only those elements that have current flowing through them for

(a) $t < 0$

(b) $0 < t < 2$ s

(c) $t > 2$ s





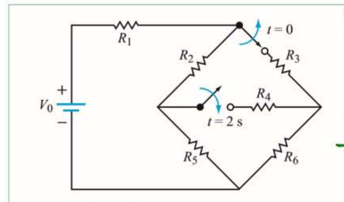


Figure P1.42: Circuit for Problem 1.42.

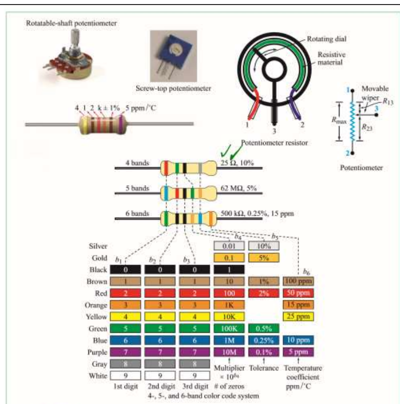


Figure 2-2: Various types of resistors. Tabular-shaped resistors usually are color-coded by 4-, 5-, or 6-band systems.

Find V_x, V_z

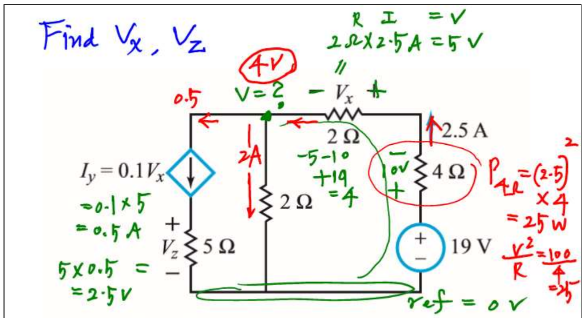


Figure P1.40: Circuit for Problem 1.40.

► The algebraic sum of the voltages around a closed loop must always be zero. ◄

This statement defines ***Kirchhoff's voltage law*** (KVL). In equation form, KVL is given by

$$\sum_{n=1}^{N} \upsilon_n = 0 \qquad \textbf{(KVL)}, \qquad (2.11)$$

where N is the total number of branches in the loop and υ_n is the nth voltage across the nth branch. Application of Eq. (2.11) requires the specification of a sign convention to use with it. Of those used in circuit analysis, the sign convention we chose to use in this book consists of two steps.

~~Hence,~~ For the loop in **Fig. 2-12**, starting at the negative terminal of the 4 V voltage source, application of Eq. (2.11) yields

KVL: $$-4 + V_1 - V_2 - 6 + V_3 - V_4 = 0. \qquad (2.12)$$

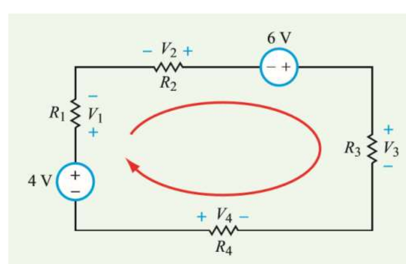


Figure 2-12: One-loop circuit.

Table 2-4: Equally valid, multiple statements of Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL).

KCL

- Sum of all currents entering a node = 0
  [i = "+" if entering; i = "−" if leaving]
- Sum of all currents leaving a node = 0
  [i = "+" if leaving; i = "−" if entering]
- Total of currents entering = Total of currents leaving

KVL

- Sum of voltages around closed loop = 0
  [υ = "+" if + side encountered first in clockwise direction]
- Total voltage rise = Total voltage drop

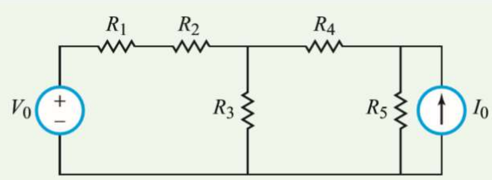
KCL/KVL Solution Recipe

- Use KCL, KVL, and Ohm's law to develop as many independent equations as the number of unknowns (N).

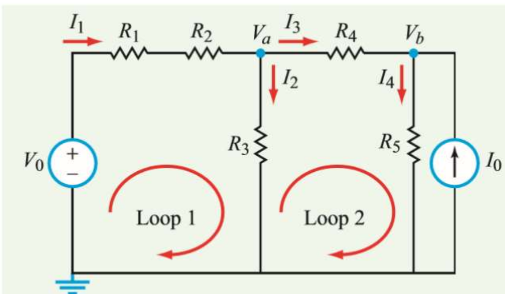
  (a) Write as many KVL loop equations as you can, picking up at least one additional circuit element for each loop. Let M be the number of such loop equations. Exclude loops that go through current sources.

  (b) Write $(N-M)$ KCL equations, making sure each node picks up an additional current.

- Write the equations in standard form (see Eq. (B.2) in Appendix B).
- Cast the standard-form equations in matrix form, as in Eqs. (B.19) and (B.20) of Appendix B.
- Apply matrix inversion to compute the values of the circuit unknowns (Appendix B).



(a) Original circuit



(b)

(b) KVL equations

The circuit contains two independent loops that do not contain the current source I_0. The associated KVL equations are:

$$-V_0 + I_1R_1 + I_1R_2 + R_3I_2 = 0 \quad \textbf{(Loop 1)},$$

$$-I_2R_3 + I_3R_4 + I_4R_5 = 0 \quad \textbf{(Loop 2)}.$$

4 unknows
2 equations

Alternatively, we can replace either of the two loop equations with the KVL equation for the perimeter loop that includes both of them, namely the loop that starts at the ground node, then goes clockwise through V_0, R_1, R_2, R_4, and R_5, and back to the ground node. Either approach leads to the same final result.

(c) KCL equations

We have two extraordinary nodes (in addition to the ground node). We designate their voltages as shown in **Fig. 2-14(b)**. With current defined as positive when entering a node, their KCL equations are

$$I_1 - I_2 - I_3 = 0 \quad \textbf{(Node } a\textbf{)},$$

$$I_3 - I_4 + I_0 = 0 \quad \textbf{(Node } b\textbf{)}.$$

additional
2 equations

(d) Arrange equations in matrix form

4 equations ⇒

$$\underbrace{\begin{bmatrix} (R_1+R_2) & R_3 & 0 & 0 \\ 0 & -R_3 & R_4 & R_5 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}} = \underbrace{\begin{bmatrix} V_0 \\ 0 \\ 0 \\ -I_0 \end{bmatrix}}_{\mathbf{B}}.$$

4 unknowns

This is in the form

$$\mathbf{AI} = \mathbf{B}.$$

(e) Matrix inversion

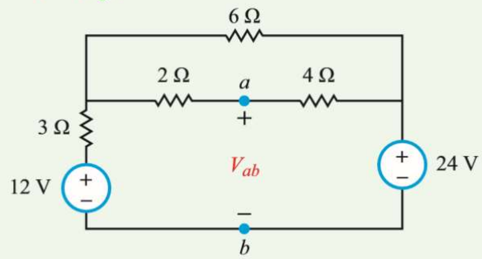
$I = A^{-1}B = \frac{\text{Adj } A}{|A|} B$

After replacing the sources and resistors with their specified numerical values, matrix reduction, per MATLAB, MathScript, or the procedure outlined in Appendix B-2, leads to

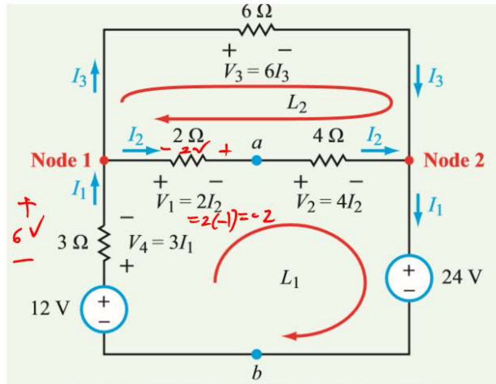
$$I_1 = 1.1\text{ A}, \quad I_2 = 0.9\text{ A},$$

$$I_3 = 0.2\text{ A}, \quad I_4 = 1\text{ A}, .$$

Find V_{ab}



(a) Given circuit



(b) After assigning currents at nodes 1 and 2

In terms of the labeled voltages, application of KVL around the two loops gives

$$-12 + V_4 + V_1 + V_2 + 24 = 0, \qquad \textbf{(KVL for Loop 1)} \quad (2.14a)$$

$$V_3 - V_2 - V_1 = 0. \qquad \textbf{(KVL for Loop 2)} \quad (2.14b)$$

Using Ohm's law for the four resistors, the two KVL equations become

$$-12 + 3I_1 + 2I_2 + 4I_2 + 24 = 0, \qquad \textbf{(KVL for Loop 1)} \quad (2.15a)$$

$$6I_3 - 4I_2 - 2I_2 = 0. \qquad \textbf{(KVL for Loop 2)} \quad (2.15b)$$

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$$-12 + V_4 + V_1 + V_2 + 24 = 0, \qquad \textbf{(KVL for Loop 1)} \quad (2.14a)$$

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$$6I_3 - 4I_2 - 2I_2 = 0. \qquad \textbf{(KVL for Loop 2)} \quad (2.15b)$$

3 unknowns
2 equations

The two simultaneous equations contain three unknowns, namely I_1 to I_3. A third equation is supplied by KCL at node 1 or node 2:

$$I_1 = I_2 + I_3. \qquad \textbf{(KCL @ node 1 or 2)} \quad (2.16)$$

1 additional eq.

Equations (2.15a), (2.15b), and (2.16) constitute 3 equations in 3 unknowns. We can solve for I_1 to I_3 either by the substitution method or by matrix inversion (Appendix B). To apply the latter, we need to cast the three equations in standard form:

$$\boxed{\begin{aligned} 3I_1 + 6I_2 \qquad &= -12, \\ -6I_2 + 6I_3 &= 0, \\ I_1 - I_2 - I_3 &= 0. \end{aligned}}$$

3 unknowns
3 equations

$$\begin{aligned} 3I_1 + 6I_2 \qquad &= -12, \\ -6I_2 + 6I_3 &= 0, \\ I_1 - I_2 - I_3 &= 0. \end{aligned}$$

In matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 0 \\ 0 & -6 & 6 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} -12 \\ 0 \\ 6 \end{bmatrix}$$

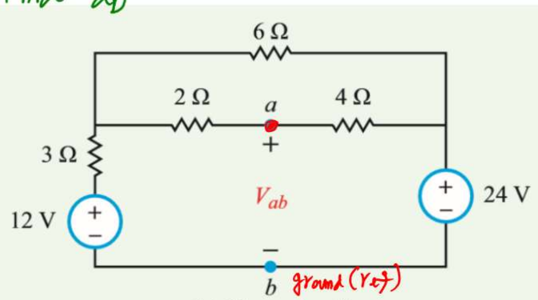
$$\begin{bmatrix} 3 & 6 & 0 \\ 0 & -6 & 6 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 0 \end{bmatrix}.$$

Matrix inversion, as outlined in Appendix B, leads to

$$I_1 = -2\text{ A}, \qquad I_2 = -1\text{ A}, \qquad I_3 = -1\text{ A}.$$

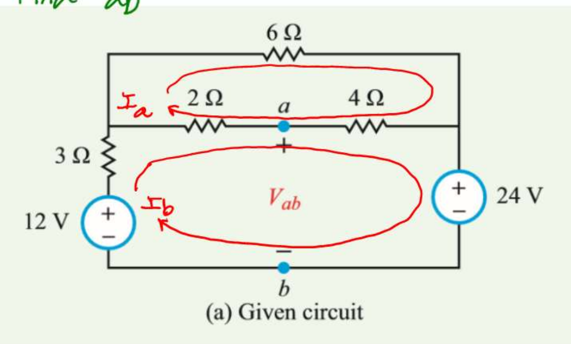
$V_2 = 4I_2 = 4(-1) = -4V \quad V_{ab} = -4 + 24 = 20\ V$

Find V_{ab} Loop Current Method



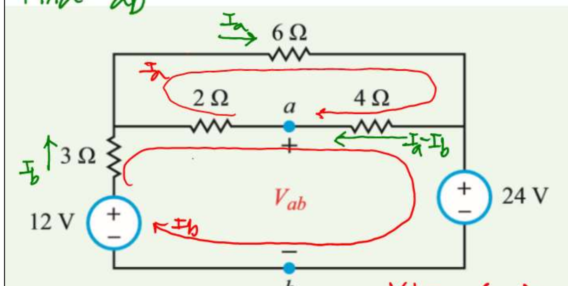
(a) Given circuit

Find V_{ab}



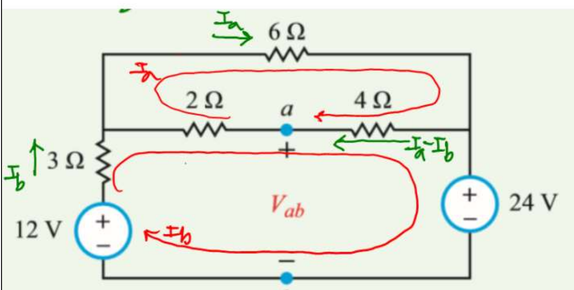
(a) Given circuit

Find V_{ab}



(a) Given circuit

$V_{ab} = -4(I_a - I_b) + 24$



KVL Loop a : $2(I_a - I_b) + 6I_a + 4(I_a - I_b) = 0$

Loop b : $-12 + 3I_b - 2(I_a - I_b) - 4(I_a - I_b) + 24 = 0$

$12I_a - 6I_b = 0$

$-6I_a + 9I_b = -12$

$\Rightarrow \quad 2I_a - I_b = 0 \quad (\Rightarrow I_b = 2I_a)$

$-2I_a + 3I_b = -4$

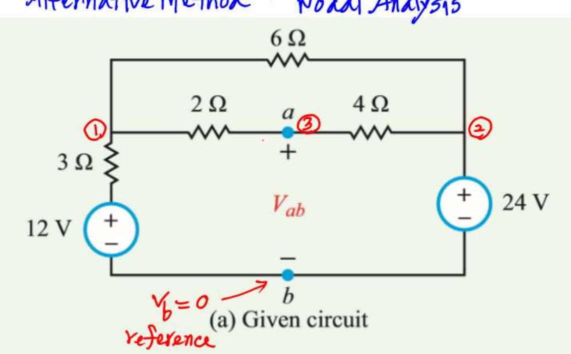
add 2 eqns $\Rightarrow 0 + 2I_b = -4 \Rightarrow I_b = -2\ A$

$2I_a = I_b \Rightarrow I_a = \frac{I_b}{2} = -1\ A$

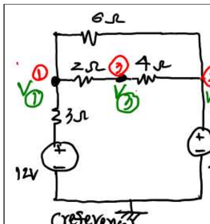
$V_{ab} = -4(I_a - I_b) + 24$

$= -4(-1 - (-2)) + 24 = 20$ ✓

Alternative Method · Nodal Analysis



(a) Given circuit



KCL

node ③ : $\frac{V_1 - V_3}{2} - \frac{V_3 - 24}{4} = 0$ (1)

node ① : $\frac{12 - V_1}{3} - \frac{V_1 - 24}{6} - \frac{V_1 - V_3}{2} = 0$ (2)

(1) $\Rightarrow 2V_1 - 2V_3 - V_3 + 24 = 0$

$\boxed{2V_1 - 3V_3 = -24}$

(2) $\Rightarrow 2(12 - V_1) - (V_1 - 24) - 3(V_1 - V_3) = 0$

$24 - 2V_1 - V_1 + 24 - 3V_1 + 3V_3 = 0$

$-6V_1 + 3V_3 = -48$

$\Rightarrow \boxed{-2V_1 + V_3 = -16}$

$\underset{\sim}{A}\,\underset{\sim}{x} = b$

$\underset{\sim}{A}^{-1}\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{A}^{-1}\underset{\sim}{b}$

$A^{-1} = \text{Adj}\,A \,/\, |A|$

$$\begin{bmatrix} 2 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix} = \begin{bmatrix} -24 \\ -16 \end{bmatrix} \Rightarrow \begin{bmatrix} V_1 \\ V_3 \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} -24 \\ -16 \end{bmatrix} = \begin{bmatrix} 18 \\ 20 \end{bmatrix}$$

$= V_{ab}$

Cramer's Rule

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \qquad AX = b$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$\begin{bmatrix} 2 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} V_{1} \\ V_{3} \end{bmatrix} = \begin{bmatrix} -24 \\ -16 \end{bmatrix}$$

$$V_{3} = \frac{\begin{vmatrix} 2 & -24 \\ -2 & -16 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ -2 & 1 \end{vmatrix}} = \frac{2 \times (-16) - (-24)(-2)}{2 \times 1 - (-3) \times (-2)} = \frac{-32-48}{-4} = \frac{-80}{-4}$$

$$= \underline{20} = V_{ab} \quad \text{ans}$$