ECE 101
Norton and Thevenin equivalent circuits

Thursday 27 October

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How do we deal with this?

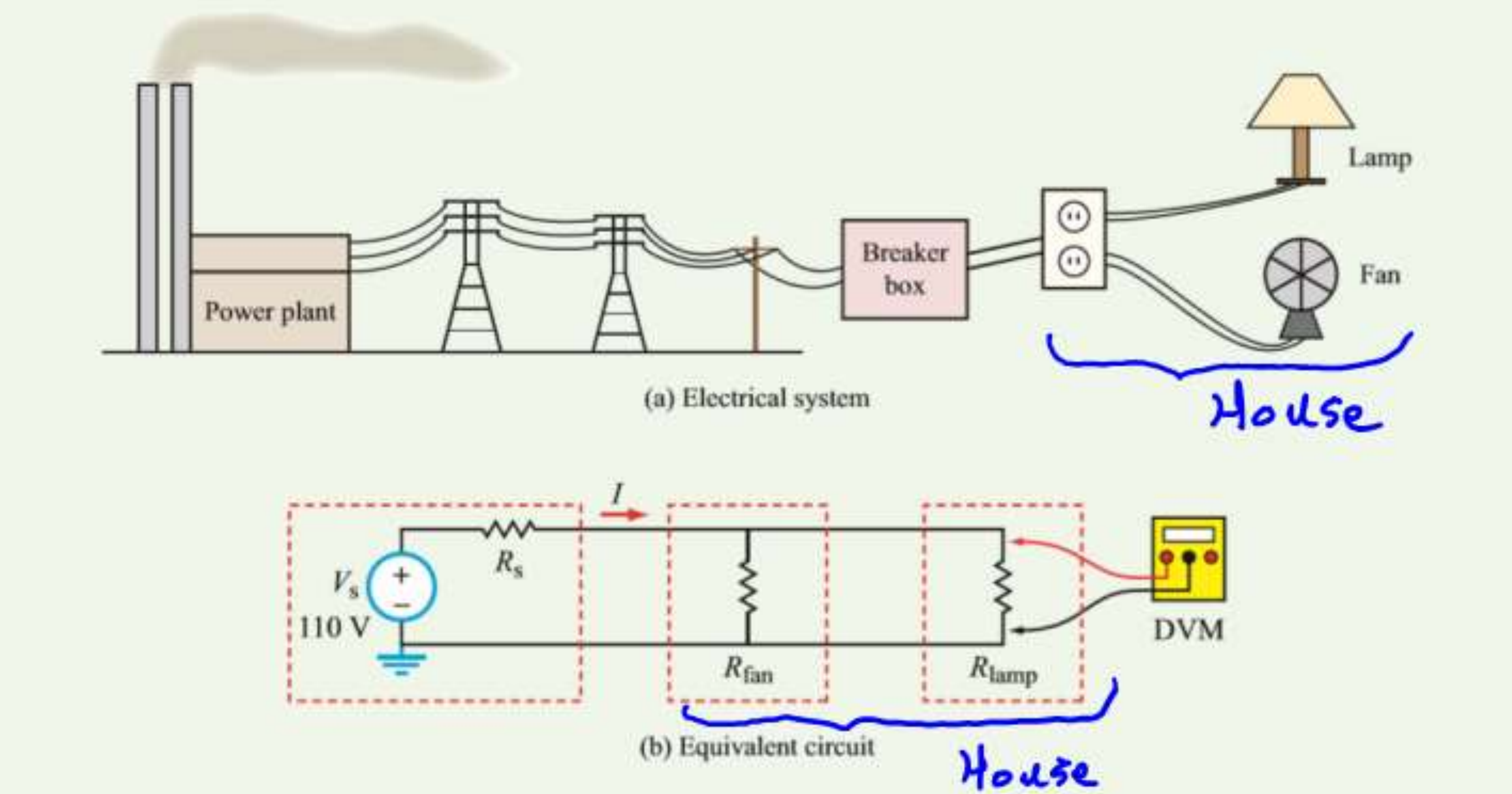


Figure 3-19: (a) Power distribution system driving a fan and a lamp in a house, and (b) block diagram of the source (power distribution system), fan, lamp, and a voltmeter measuring the voltage in the outlet.

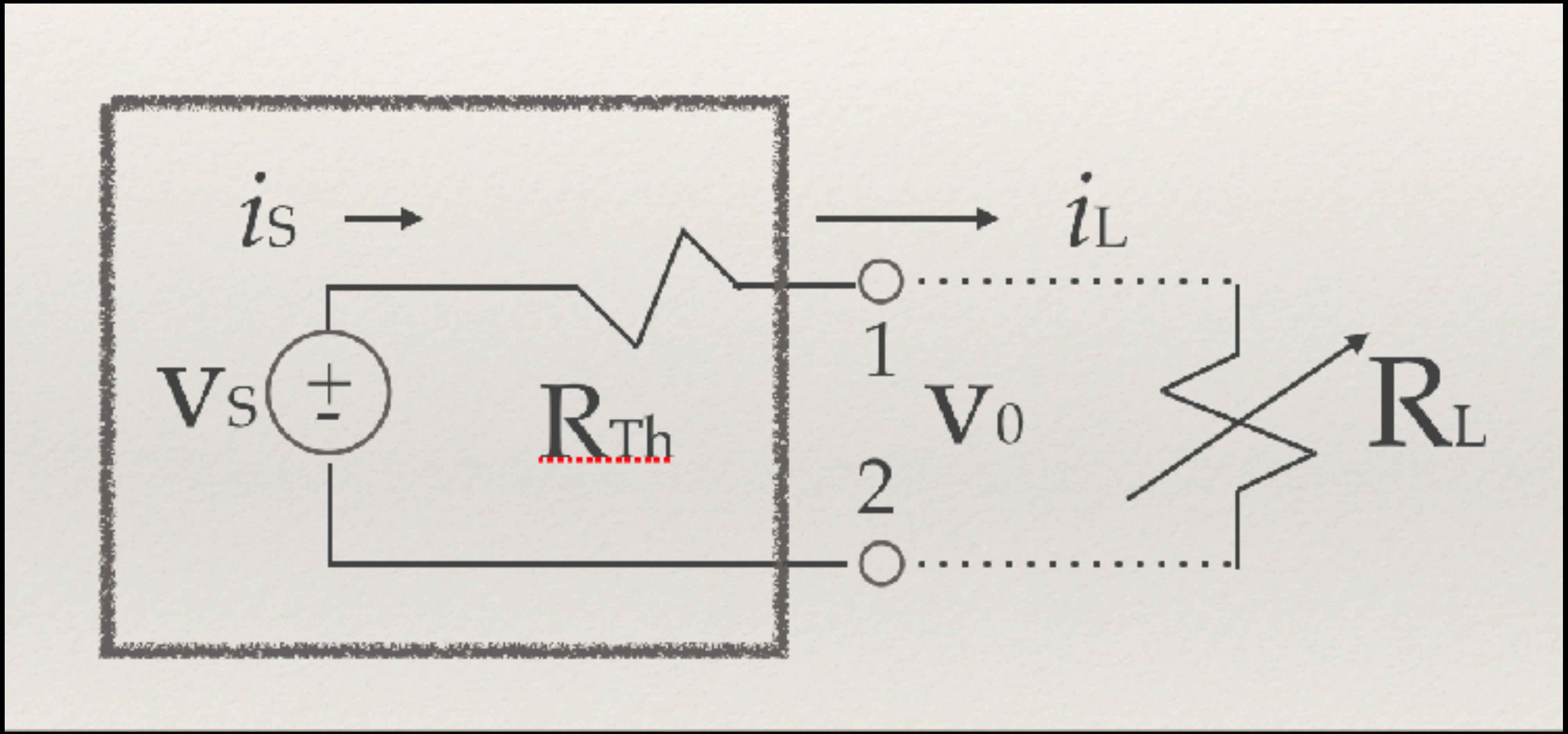
Dealing with Variable Load

- In many circuits, one element will be **variable**
- In the example of mains power: many different appliances may be plugged into an outlet, each presenting a different resistance. This variable element is called the **load**
- Ordinarily one would have to reanalyze the circuit for each change in the load…

Thevenin’s Theorem

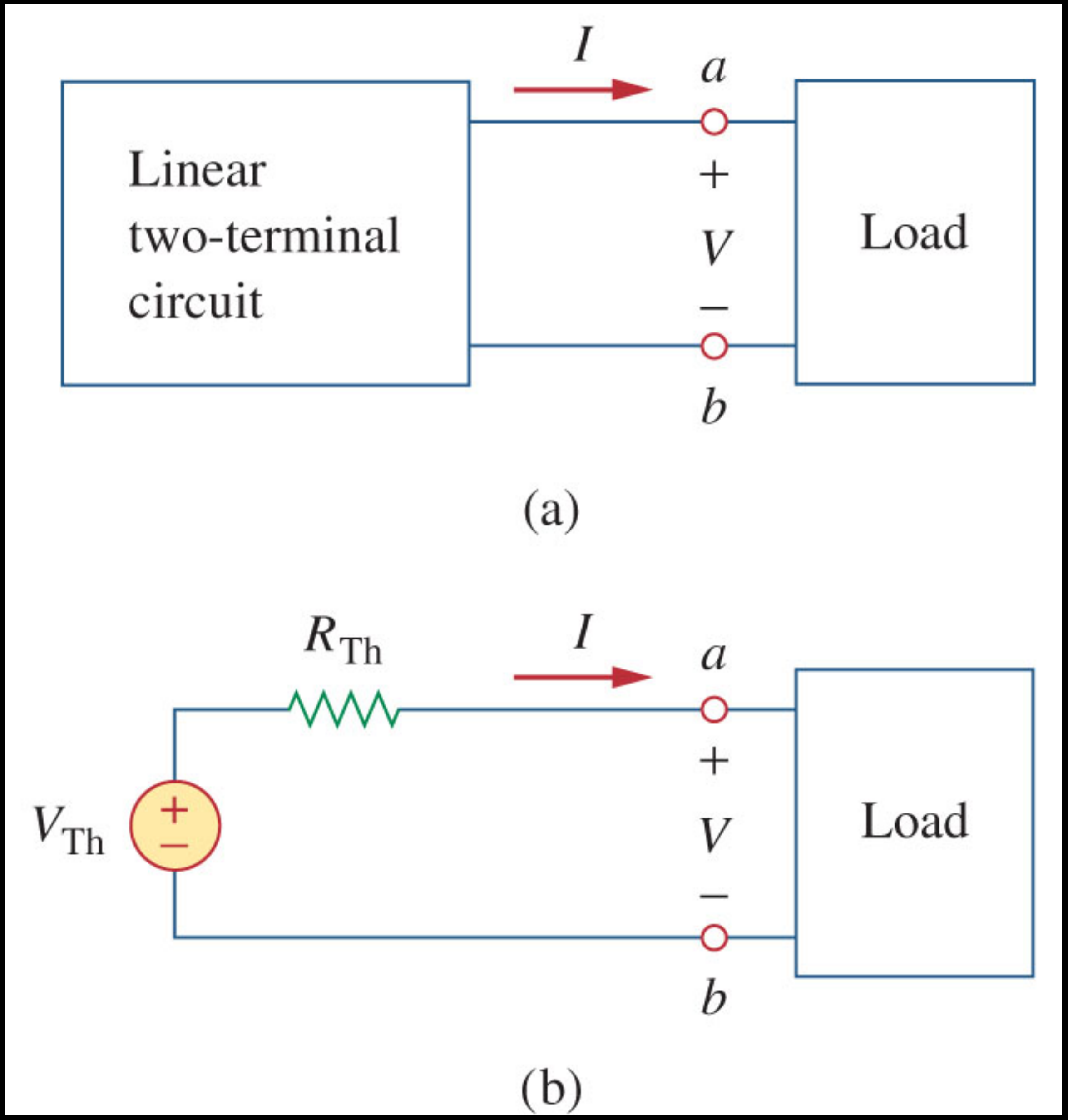
- Thevenin’s theorem allows us to simplify a large circuit and replaced it with a single independent voltage source and a single resistor.
- The equivalent circuit behaves externally as the original circuit.

Thevenin circuit



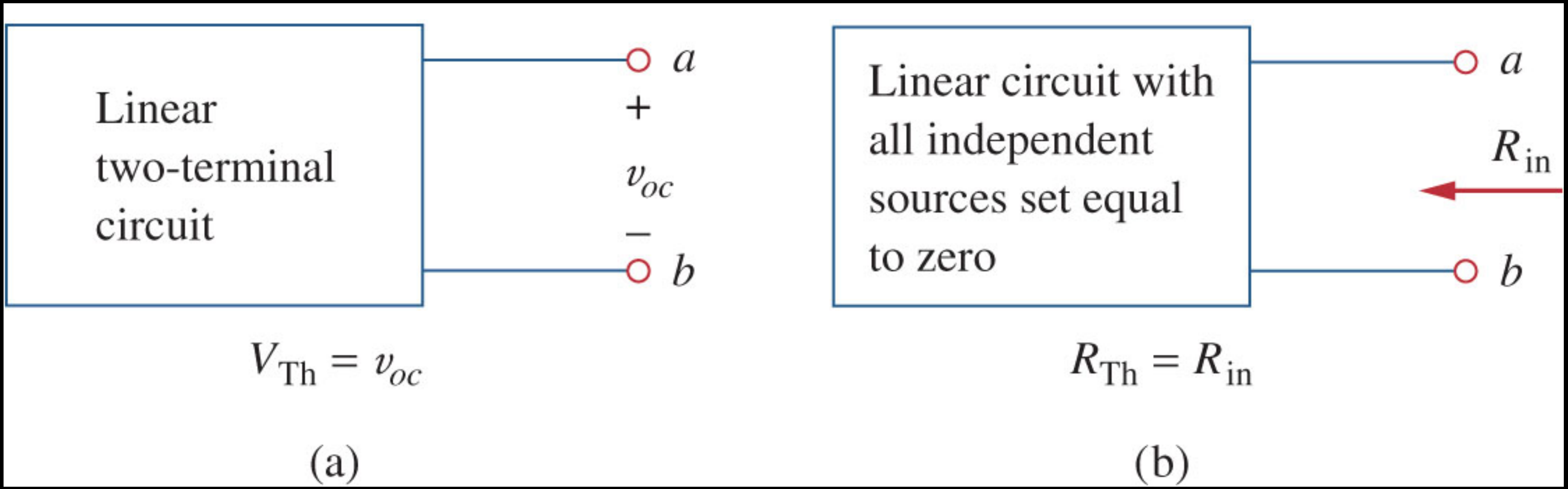
Thevenin’s Theorem

- Thevenin’s theorem states that a linear two terminal circuit may be replaced with **a voltage source and resistor**
- The voltage source’s value is equal to the open circuit voltage at the terminals
- The resistance is equal to the resistance measured at the terminals when the independent sources are turned off.



Finding R_{Th} for Thevenin

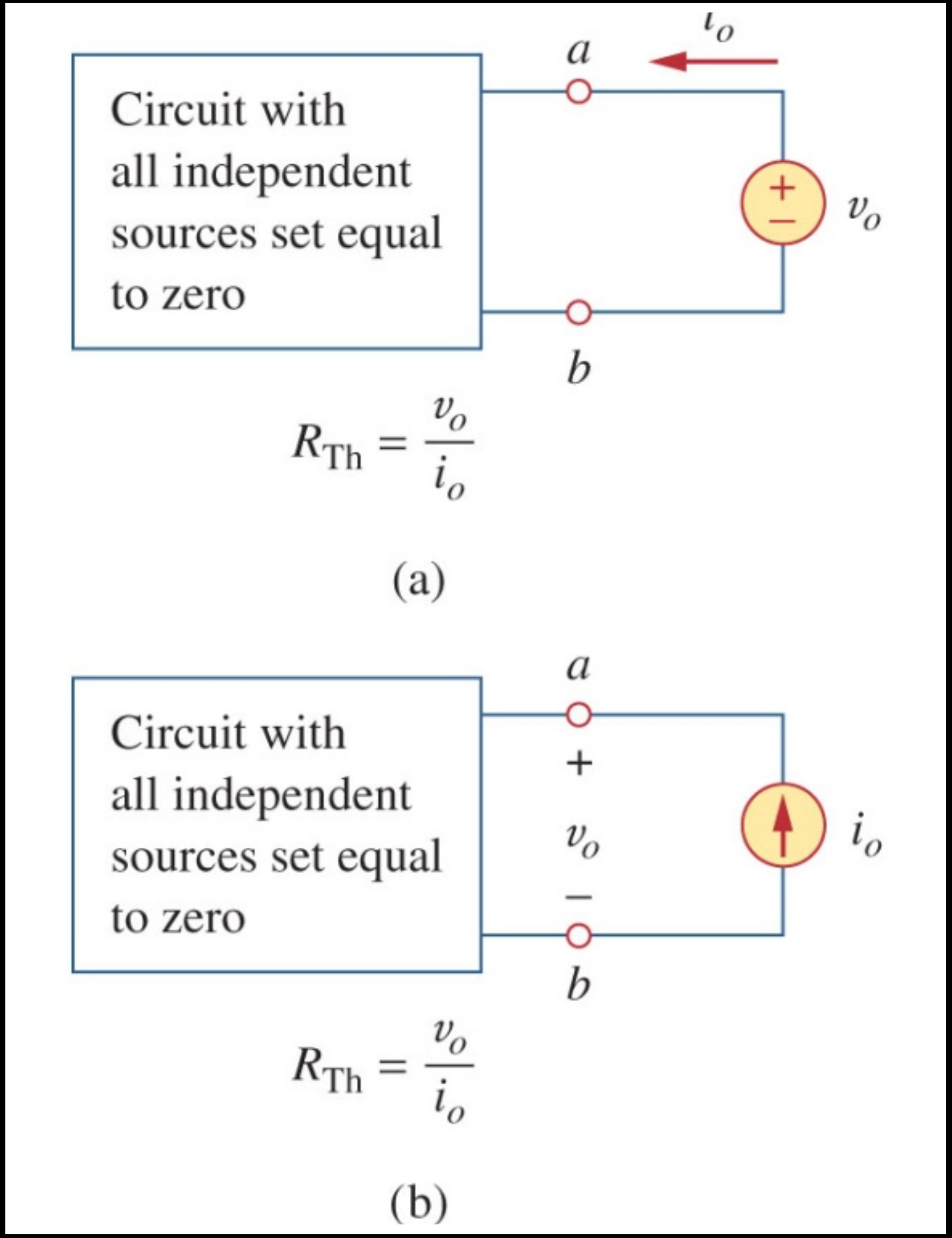
Case 1:
If there are **no dependent sources** the resistance R_{Th} may be found by simply turning off all the sources:



Finding R_{eq} for Thevenin

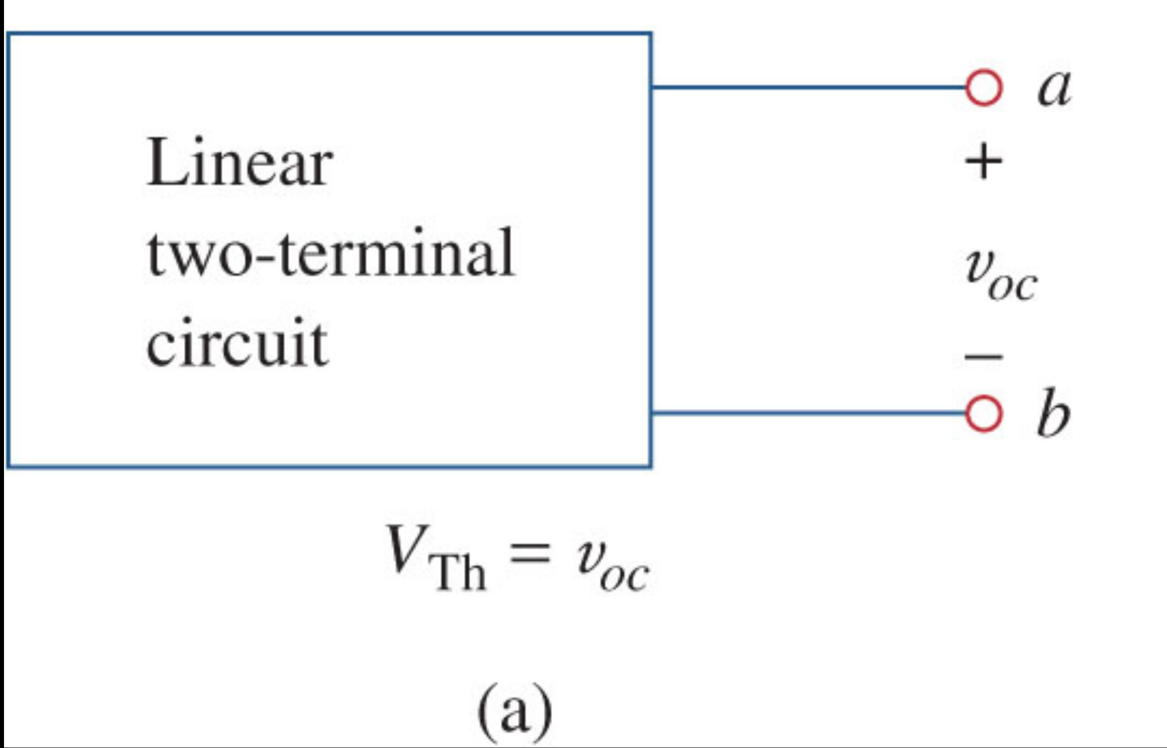
Case 2:
If there are **dependent sources:** to find the resistance R_{Th} we

- Turn off all the sources (like before, and then…)
- Apply a voltage v_0 (or current i_0) to the terminals and then determine the current i_0 (or voltage v_0)



Finding V_{Th} for Thevenin

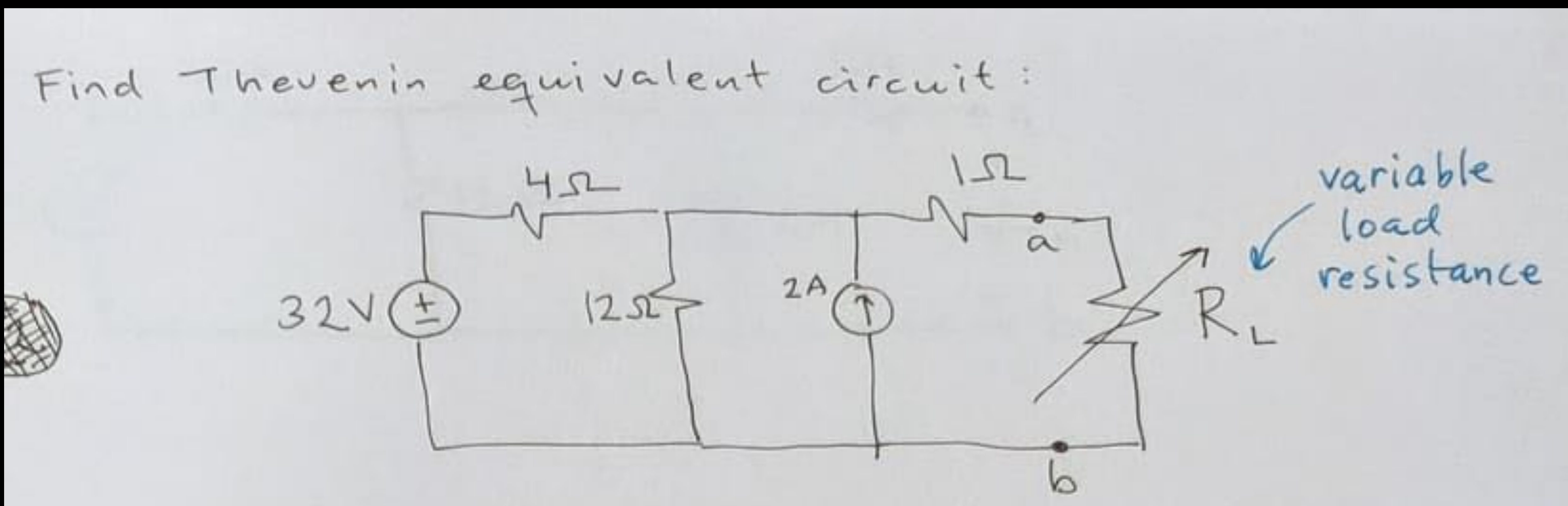
- V_{Th} (voltage source's value) is equal to the open circuit voltage at the terminals

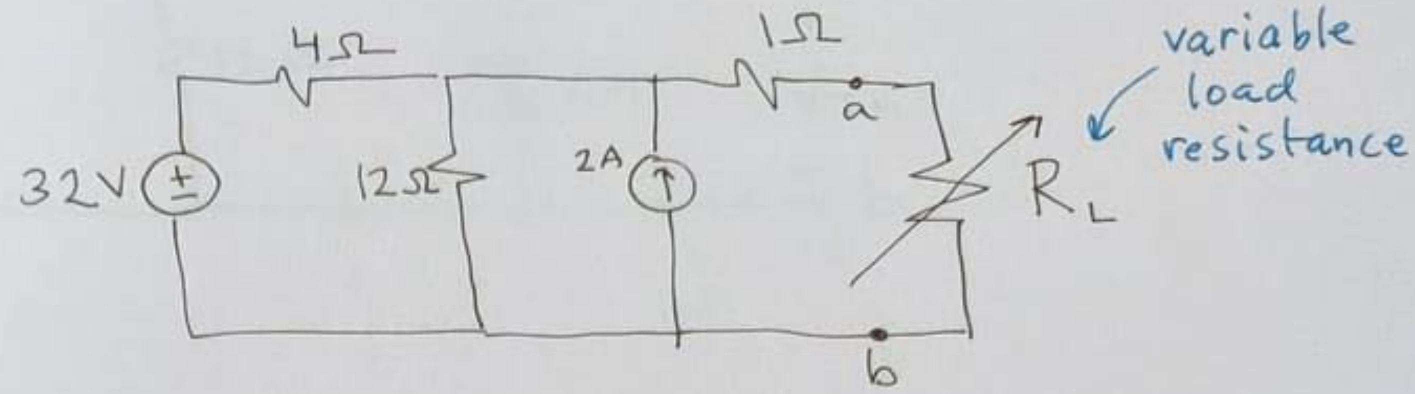


Negative Resistance?

- If we find a **negative resistance** this implies the circuit is **supplying power**
- This is reasonable with dependent sources

Example of Thevenin equivalent circuit

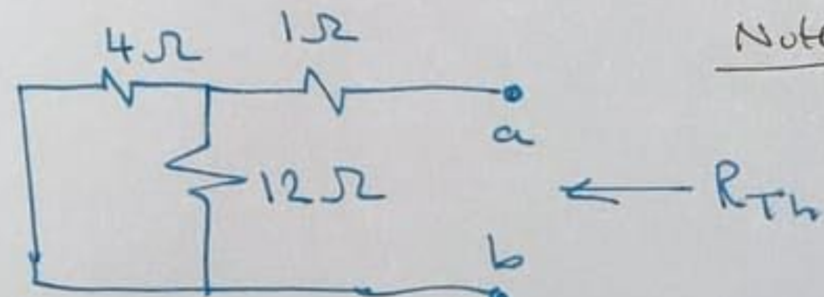




To find R_{Th}: (Thevenin equiv. resistance at terminal a b)

we find the resistance of the circuit at the terminal ab when ab is <u>open</u> and <u>all independent sources are turned off</u>.

Redraw the circuit:



Note: we turn off the 32V voltage source. Ideal voltage source has zero internal resistance so we replace it with short circuit.

We turn off the 2 A current source. Ideal current source has ∞ infinite internal resistance so we replace with open circuit.

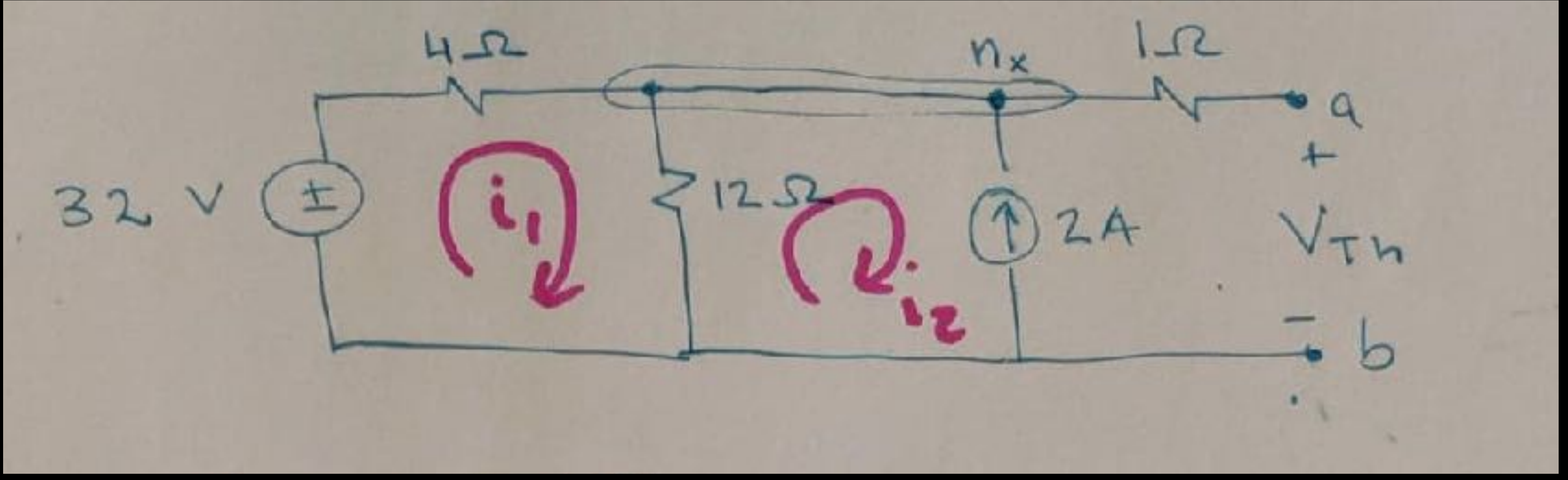
We have 4Ω and 12 Ω in parallel and this in series with 1Ω ⇒

$$R_{Th} = 4 \| 12 + 1 = \frac{4 \cdot 12}{\underbrace{4+12}_{16}} + 1 =$$

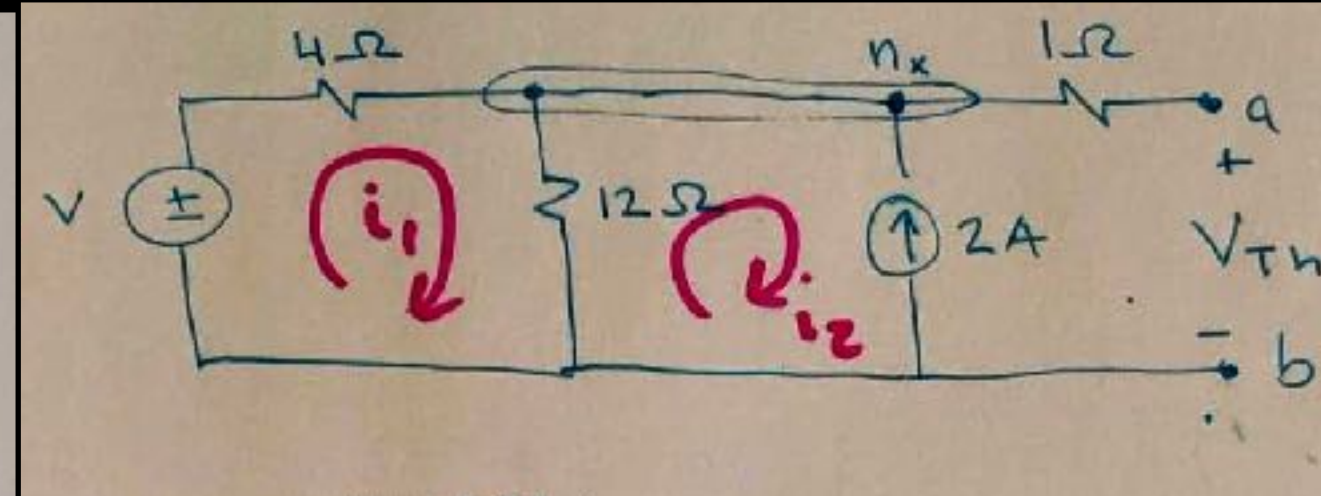
$$= \frac{\cancel{4} \cdot \cancel{4} \cdot 3}{\cancel{4} \cdot \cancel{4}} + 1 = 3 + 1 = \underline{\underline{4\ [\Omega]}}$$

- We have found R_{Th}, now we need to find V_{Th} which is the open circuit voltage at the a b terminal

- We can for example do this using mesh analysis.
Set up KVL at the loops 1 and 2:



MESH ANALYSIS :



$$\begin{cases} -32 + 4 i_1 + 12 (i_1 - i_2) = 0 \\ i_2 = -2 [A] \end{cases}$$

$$-32 + 4 \cdot i_1 + 12 (i_1 + 2) =$$

$$-32 + 4 i_1 + 12 i_1 + 24 =$$

$$= 16 i_1 - 8$$

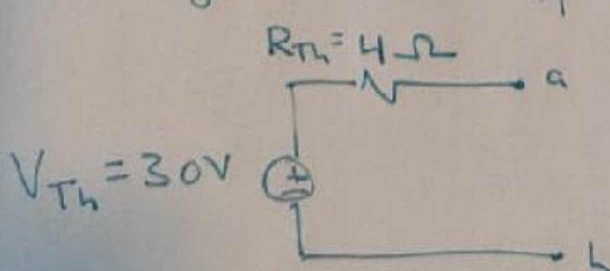
$$\Rightarrow i_1 = + \frac{8}{16} = \frac{1}{2} \ [A]$$

Voltage at node n_x is same as V_{Th} since there is no current flowing across the 1Ω resistor.

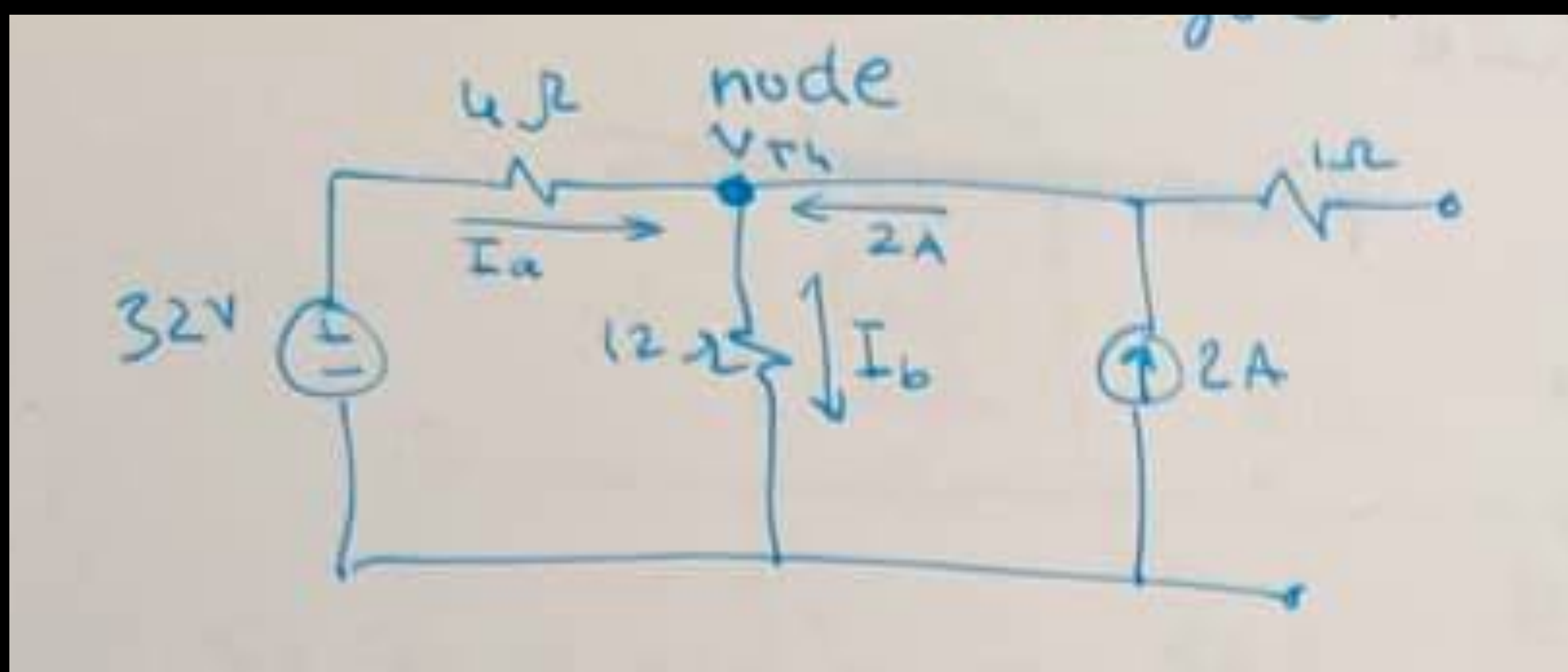
$$\Rightarrow V_{Th} = 12 (i_1 - i_2) = 12 (0.5 - (-2)) = 12 \times 2.5 = 30 [V]$$

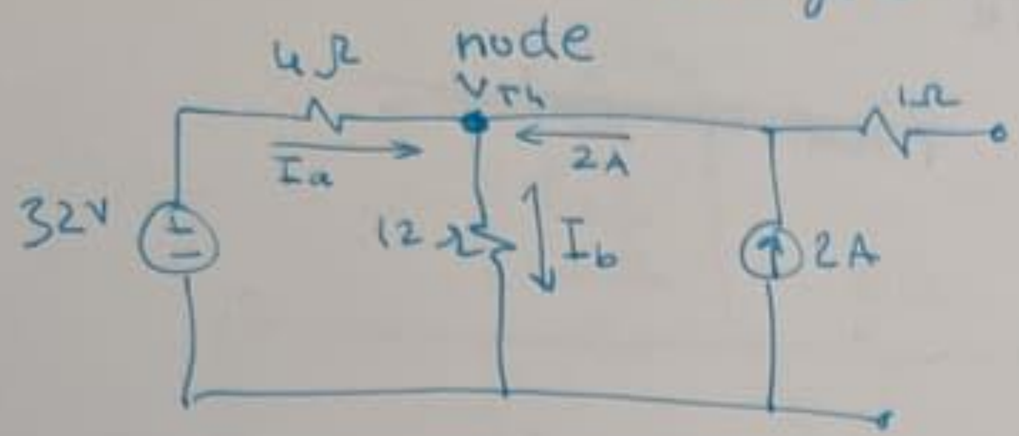
voltage drop across 12Ω resistor

We get the Thevenin equivalent circuit:



- Alternatively, we can find V_{Th} using nodal analysis:





$$\underset{\uparrow\ \frac{32-V_{Th}}{4}}{I_a} + 2 = \underset{\uparrow\ \frac{V_{th}}{12}}{I_b} \Rightarrow \frac{32-V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

$$\frac{32}{4} - \frac{V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

$$\frac{32}{4} + 2 = \frac{V_{Th}\cdot 3}{4\cdot 3} + \frac{V_{Th}}{12} = \frac{\cancel{4}\,V_{Th}}{\cancel{4}\cdot 3}$$

$$\frac{\cancel{4}\cdot 8}{\cancel{4}} + 2 = \frac{V_{Th}}{3}$$

$$10 = \frac{V_{Th}}{3}$$

$$\underline{\underline{V_{Th} = 30\,V}}$$

- Nice! We get the same answer with this method

Now what happens if we load this circuit with R_{load} = 6 Ohm ? or with R_{load} = 16 Ohm?

Ex: find out what happens when $R_{Load} = 6$:

Current through R_L is :

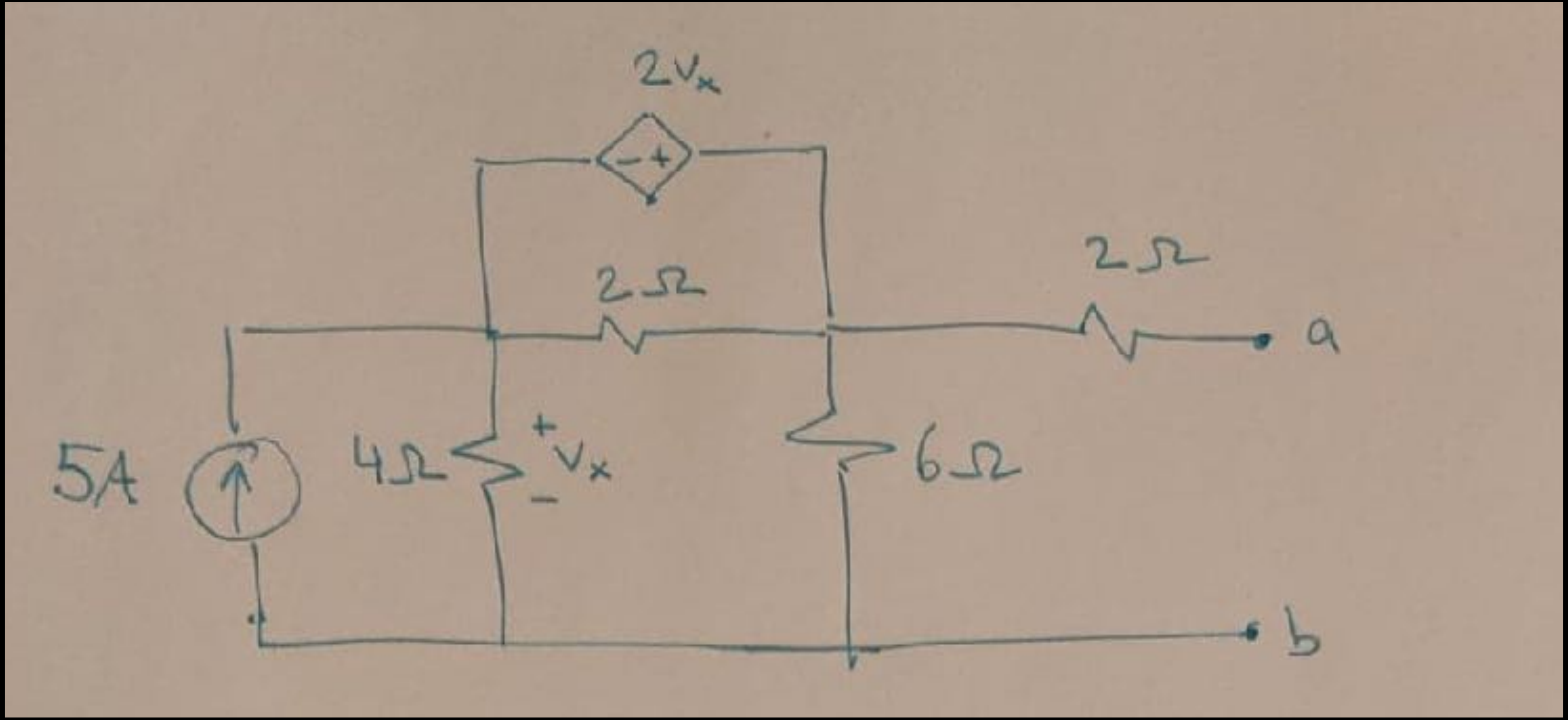
$$I_L = \frac{V_{Th}}{R_{Th}+R_L} = \frac{30}{4+R_L} =$$

$$= \frac{30}{10} = \underline{\underline{3A}}$$

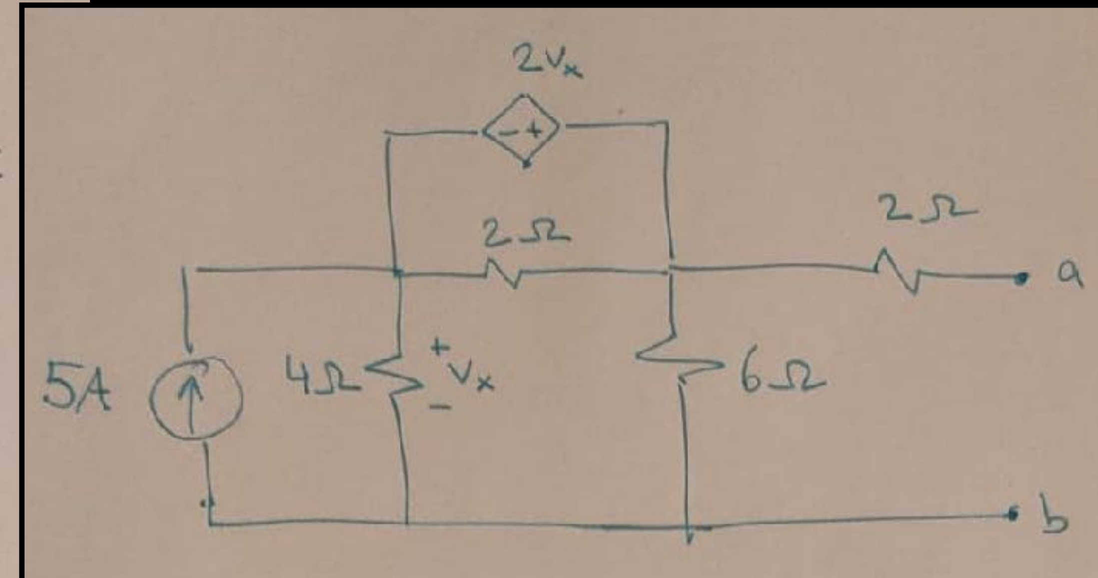
what if load has resistance 16 Ω ?

$$I_L = \frac{30}{4+R_L} = \frac{30}{4+16} = \frac{30}{20} = \underline{\underline{1.5A}}$$

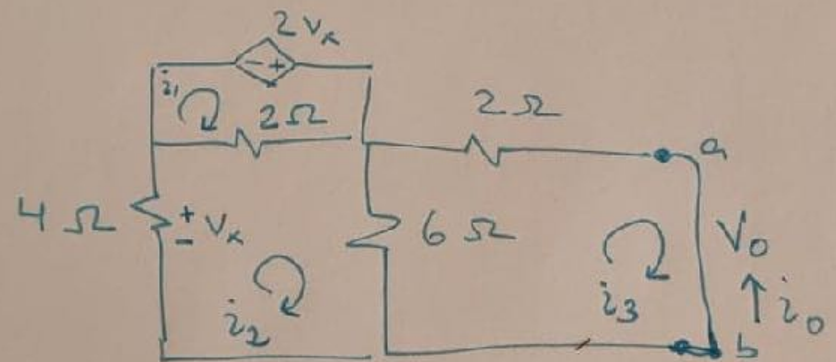
Example of finding Thevenin equivalent for circuit with dependent sources



- Turn off all independent sources
- Connect a source V_o to ab terminal
- Find current i_o at terminal to obtain R_{Th} :



5A current source is turned off, acts like an open circuit! $\Rightarrow$



Easy way to handle this, set $V_o = 1\,V$ (cancels out anyway)

For example we can use mesh analysis: KVL:

Kirchhoffs Voltage Law

Loop 1:

$-2V_x + 2(i_1 - i_2) = 0 \Rightarrow$

$\Rightarrow V_x = i_1 - i_2$

Voltage drop over 4Ω resistor :

$V_x = 4(-i_2) = -4i_2$

passive sign convention: negative since current flows into negative terminal

$i_1 - i_2 = -4i_2 \Rightarrow$

$i_1 = -3i_2$

Loop 2:

$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$

Loop 3:

$6(i_3 - i_2) + 2i_3 + 1 = 0$

For example we can use mesh analysis: KVL: {Kirchhoffs Voltage Law}

Loop 1:

$-2V_x + 2(i_1 - i_2) = 0 \Rightarrow$

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Loop 2:

$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$

Loop 3:

$6(i_3 - i_2) + 2i_3 + 1 = 0$

Eq 1:

$4i_2 + 2i_2 - 2i_1 + 6i_2 - 6i_3 = 0$

$= i_2(4+2+6) - 2i_1 - 6i_3 =$

$= -2i_1 + 12i_2 - 6i_3 = 0$

we have that $i_1 = -3i_2 \Rightarrow$

$-2(-3i_2) + 12i_2 - 6i_3 =$

$\Rightarrow 18i_2 = 6i_3 \Rightarrow 3i_2 = i_3$

Eq 2: $6i_3 - 6i_2 + 2i_3 + 1 = 0$

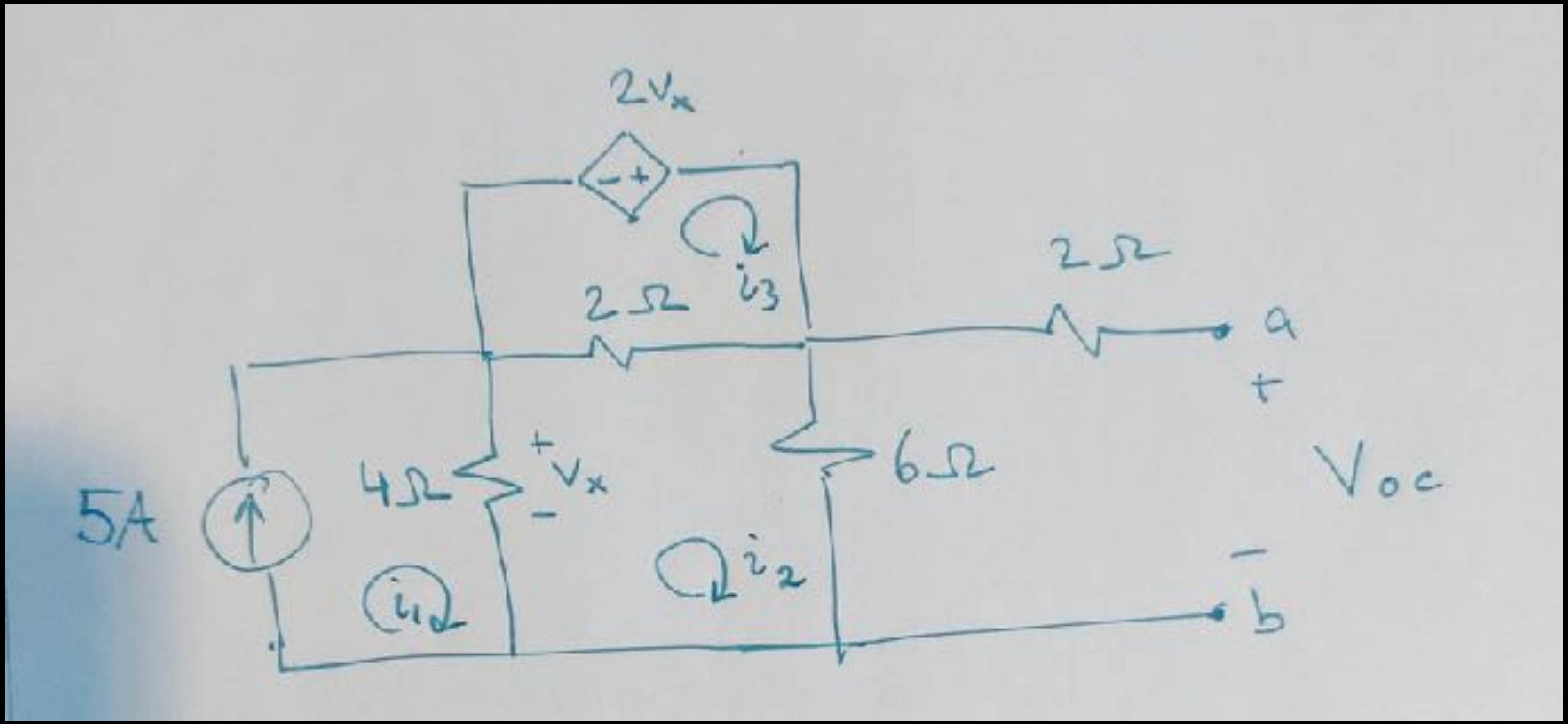
$8i_3 - 6i_2 = -1$ from (1) we have $i_2 = \frac{i_3}{3}$

$\Rightarrow 8i_3 - \cancel{6}^{2}\left(\frac{i_3}{\cancel{3}}\right) = -1$ $6i_3 = -1$ $i_3 = -\frac{1}{6}$ [A]

we see that: $i_0 = -i_3 = \frac{1}{6}$ [A] $\Rightarrow R_{Th} = \frac{1V}{i_0} = 6\Omega$

- We have found R_{Th}, now we need to find V_{Th} which is the open circuit voltage V_{oc} at the a b terminal

- We can for example do this using mesh analysis.
  Set up KVL at the loops 1, 2 and 3:



We see that $4(i_1 - i_2) = V_x$

Loop 1 : $i_1 = 5$

Loop 3 : $-2v_x + 2(i_3 - i_2) = 0 \Rightarrow$

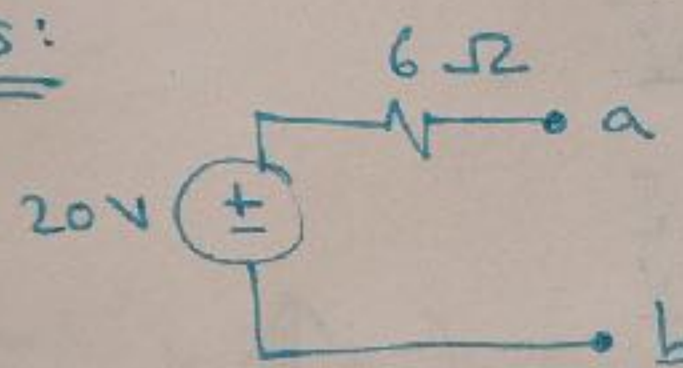
$\Rightarrow V_x = i_3 - i_2 \quad \cdots (1)$

Loop 2 : $4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$

Solve for $i_2 = \frac{10}{3}$ [A]

$\Rightarrow V_{Th} = V_{oc} = 6i_2 = 20V$

Ans:



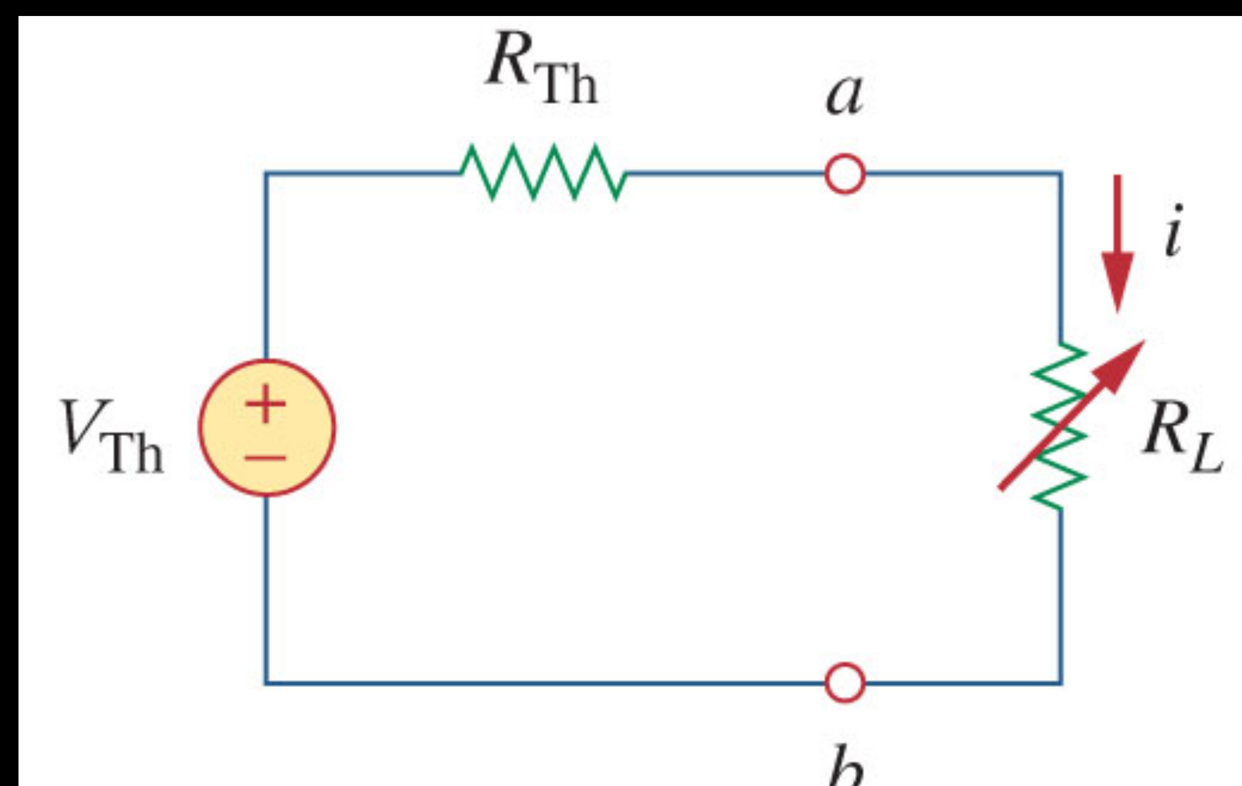
Maximum Power Transfer

- In many applications, a circuit is designed to power a load
- Among those applications there are many cases where we **wish to maximize the power transferred to the load**
- Unlike an ideal source, internal resistance will restrict the conditions where maximum power is transferred.

Finding Maximum Power Transfer using Thevenin equivalent

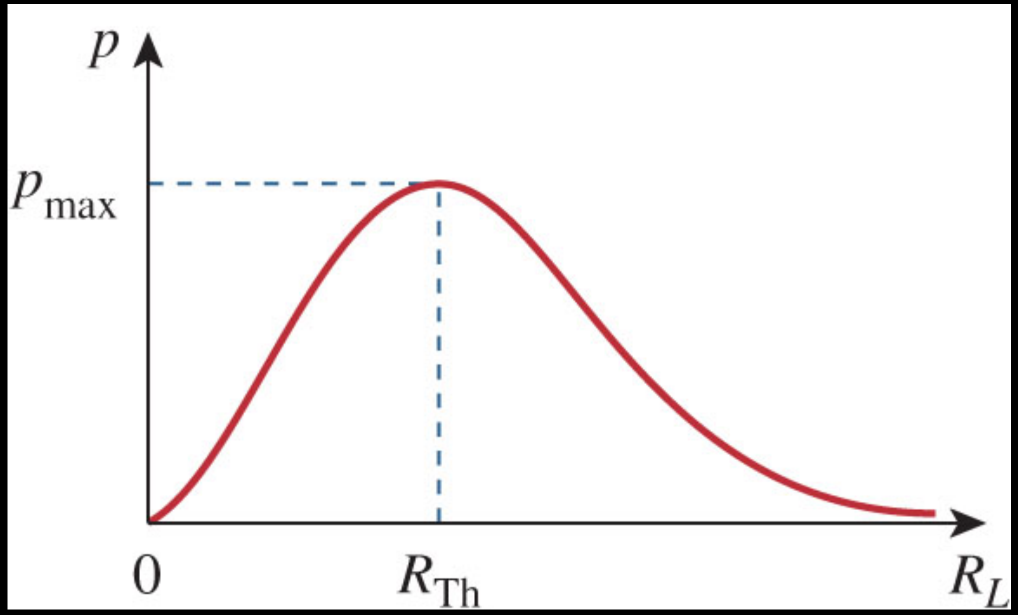
- We can use the Thevenin equivalent circuit for finding the maximum power in a linear circuit
- We will assume that the load resistance can be varied
- Looking at the equivalent circuit with load included, the power transferred is:
  ($P = IV = I^2V$)

$$p = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

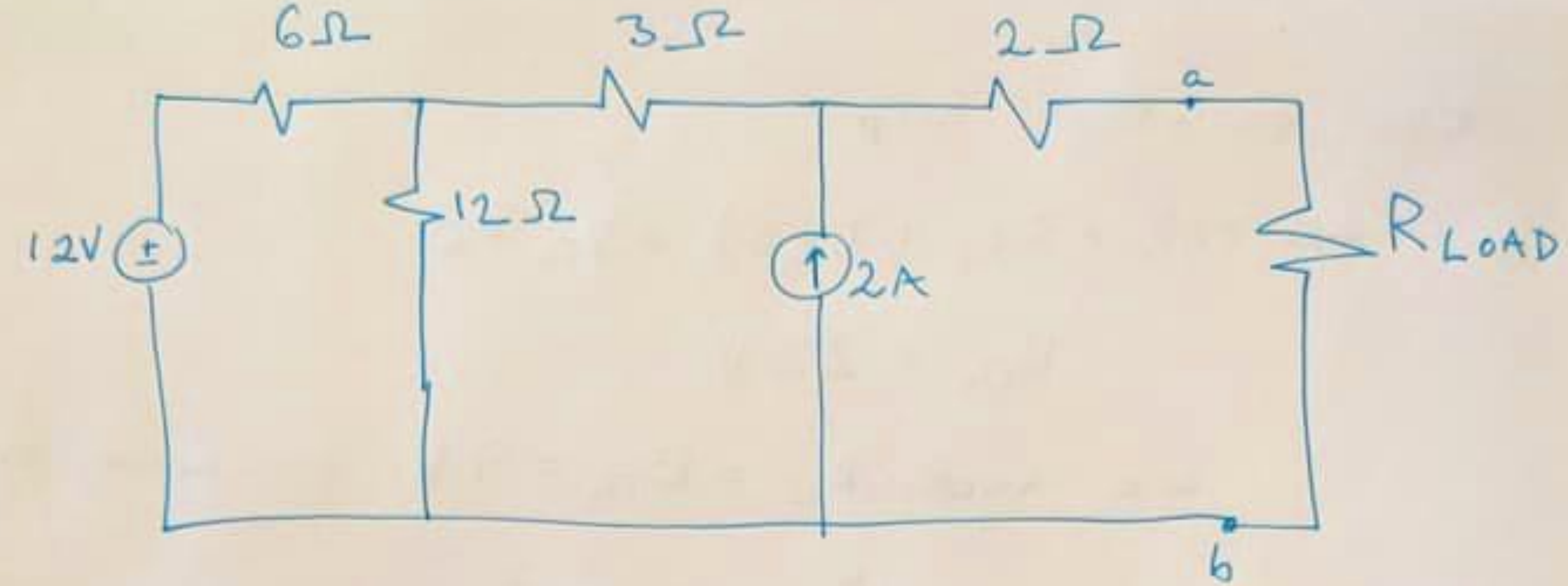


Maximum Power Transfer

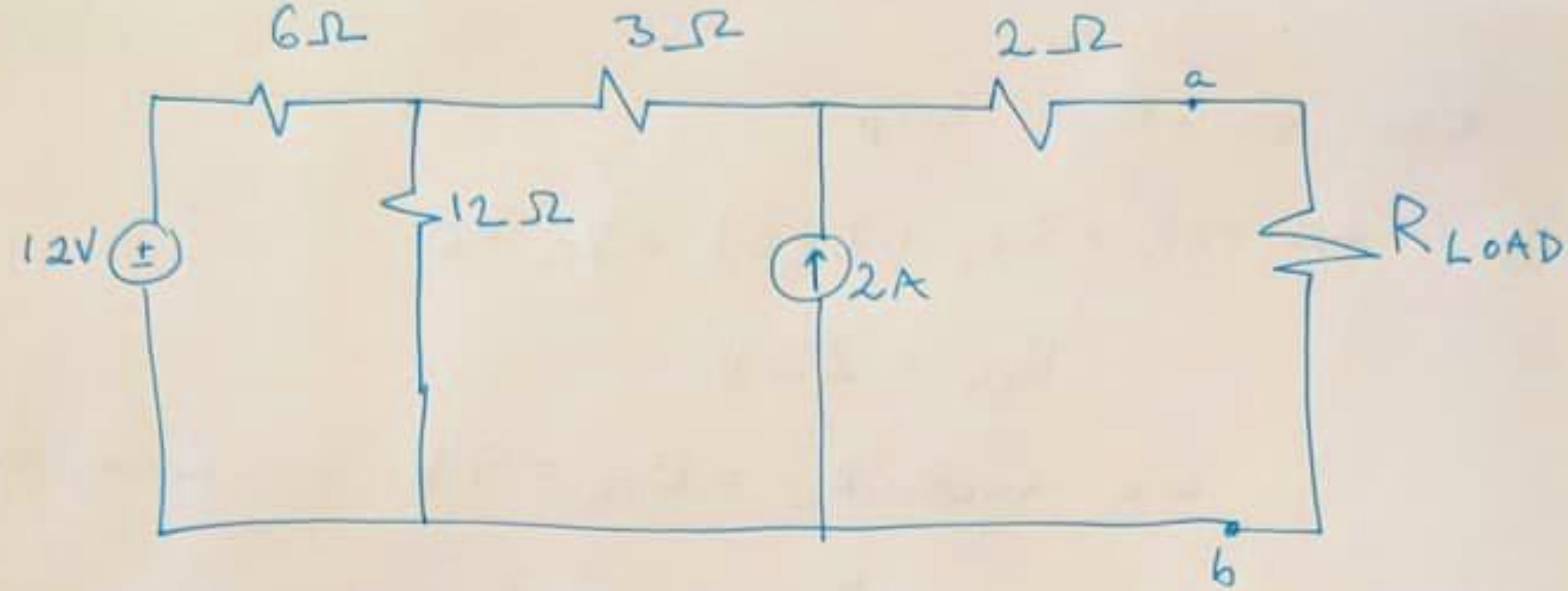
- For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as shown
- As R_L approaches 0 and ∞ the power transferred goes to zero.
- Maximum power is transferred when R_L=R_{Th}



Example
Max Power Transfer:



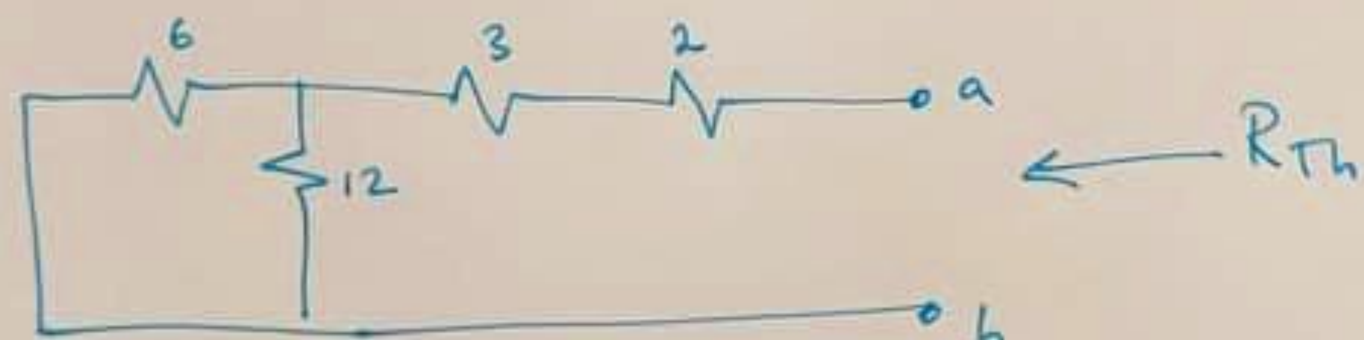
FIND R_L FOR MAX POWER TRANSFER

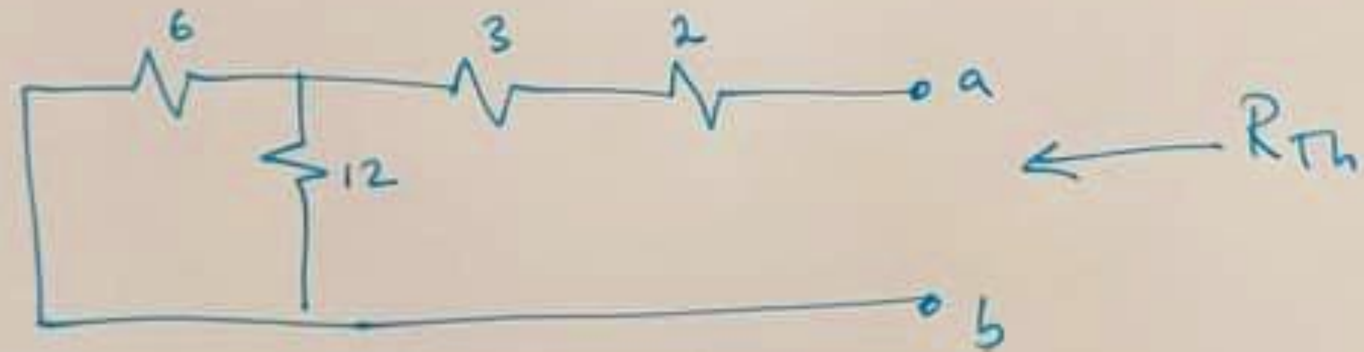


FIND R_L FOR MAX POWER TRANSFER

- Find R_{Th} : Turn off all the sources

(Remember: voltage source gets shorted
current source: open circuit)





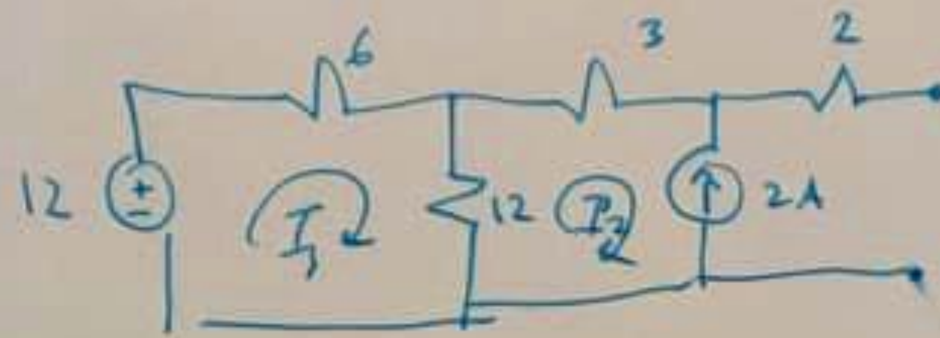
Resistors in series & parallel:

$$6 \,||\, 12 + 3 + 2 = \underbrace{\frac{6\cdot 12}{6+12}} + 5 = \underline{\underline{9\ [\Omega]}}$$

$$\frac{\cancel{6}\cdot 12}{18} = 4$$

$$3\cdot\cancel{6}$$

Now find V_{Th}: Open circuit



$12 i_1 = 6 i_1 + 12(i_1 - i_2)$ and $i_2 = 2A$

$\Rightarrow i_1 = \frac{-2}{3}$ [A]

KVL on other loop:

$$-12 + 6 i_1 + 3 i_2 + 2(0) + V_{Th} = 0$$

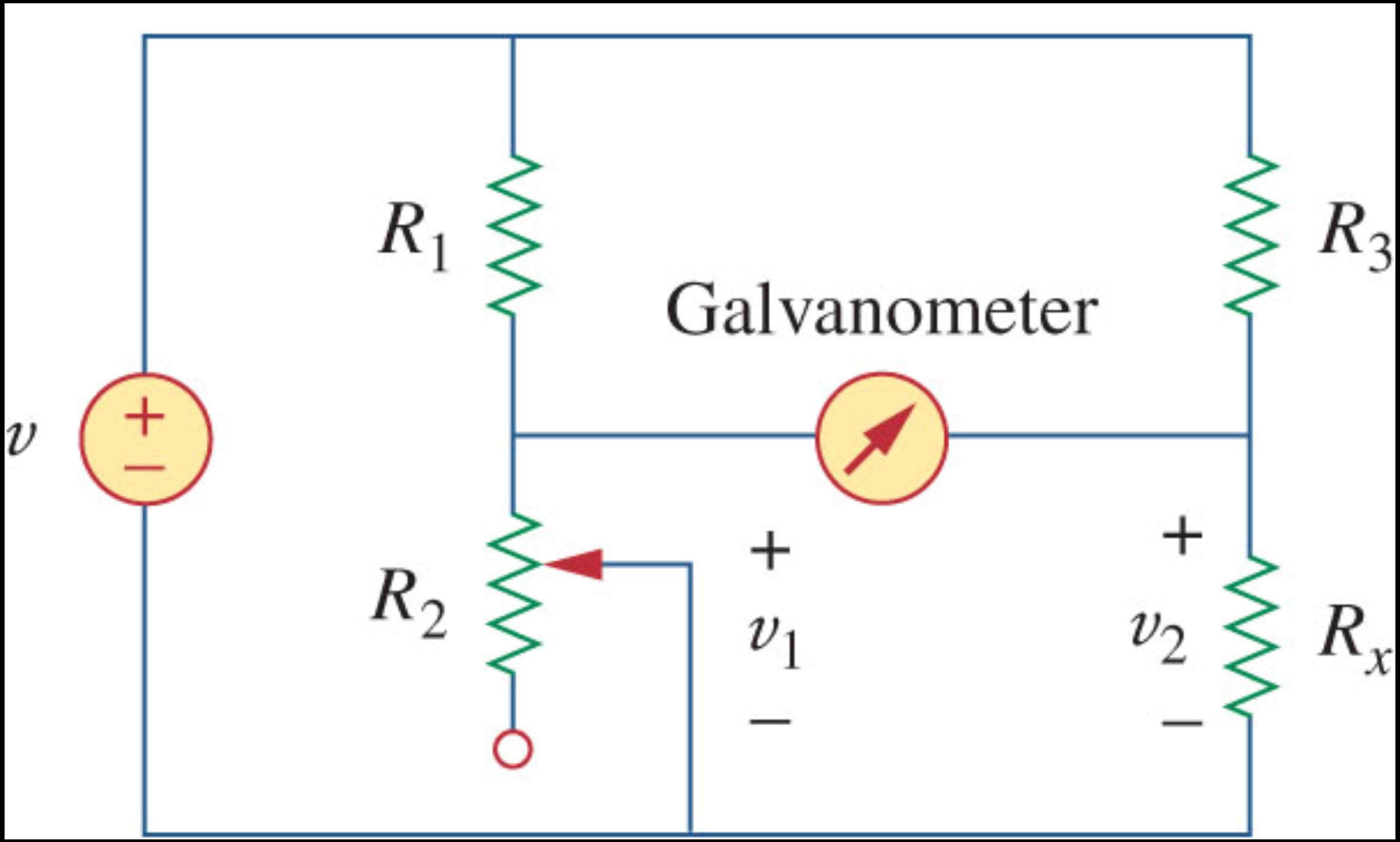
$$V_{Th} = 22\,V$$

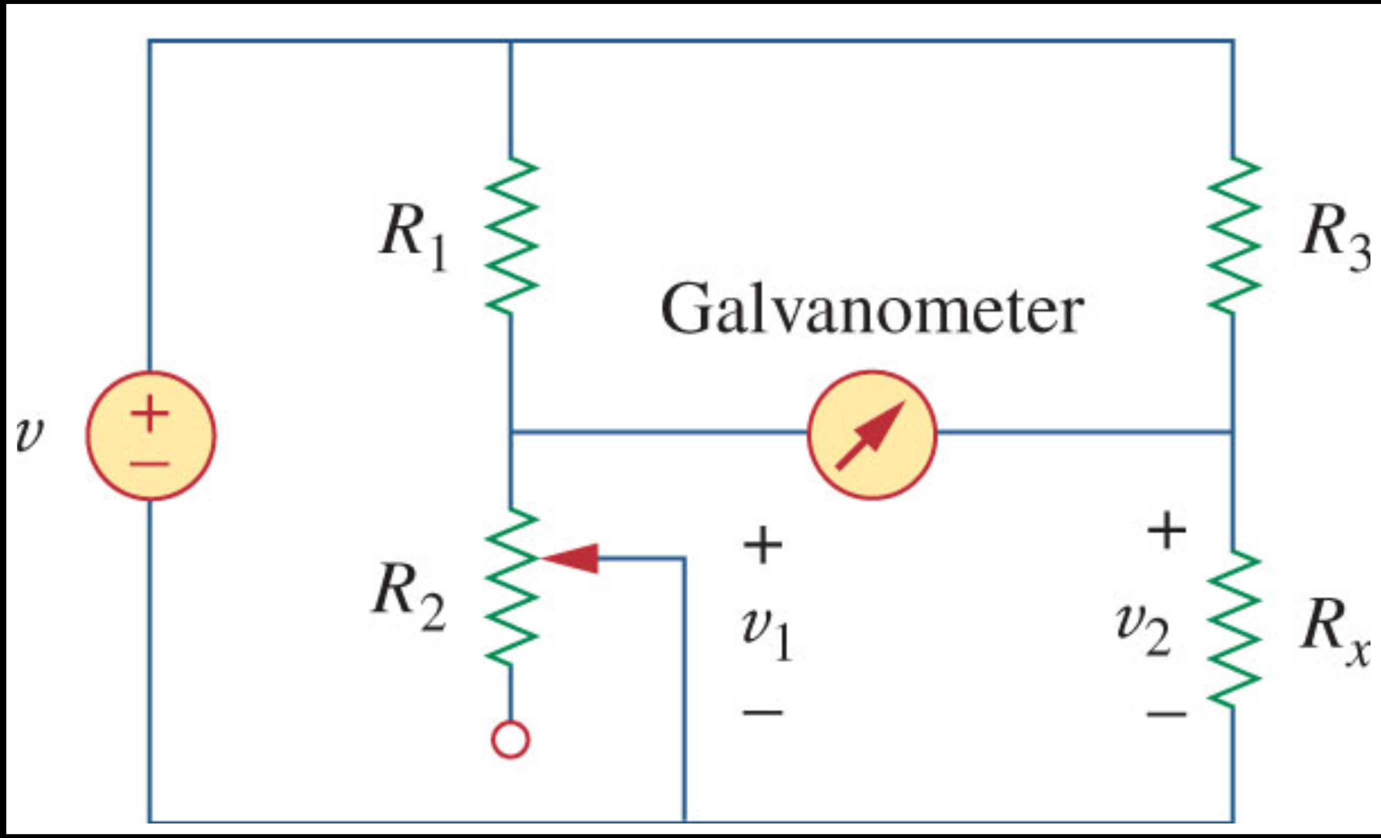
we have $R_L = R_{Th} = 9\,V$ for max power

$$P_{max} = \frac{V_{Th}^2}{4 R_L} = \frac{22^2}{4 \times 9} = 13.44\ [W]$$

Wheatstone bridge

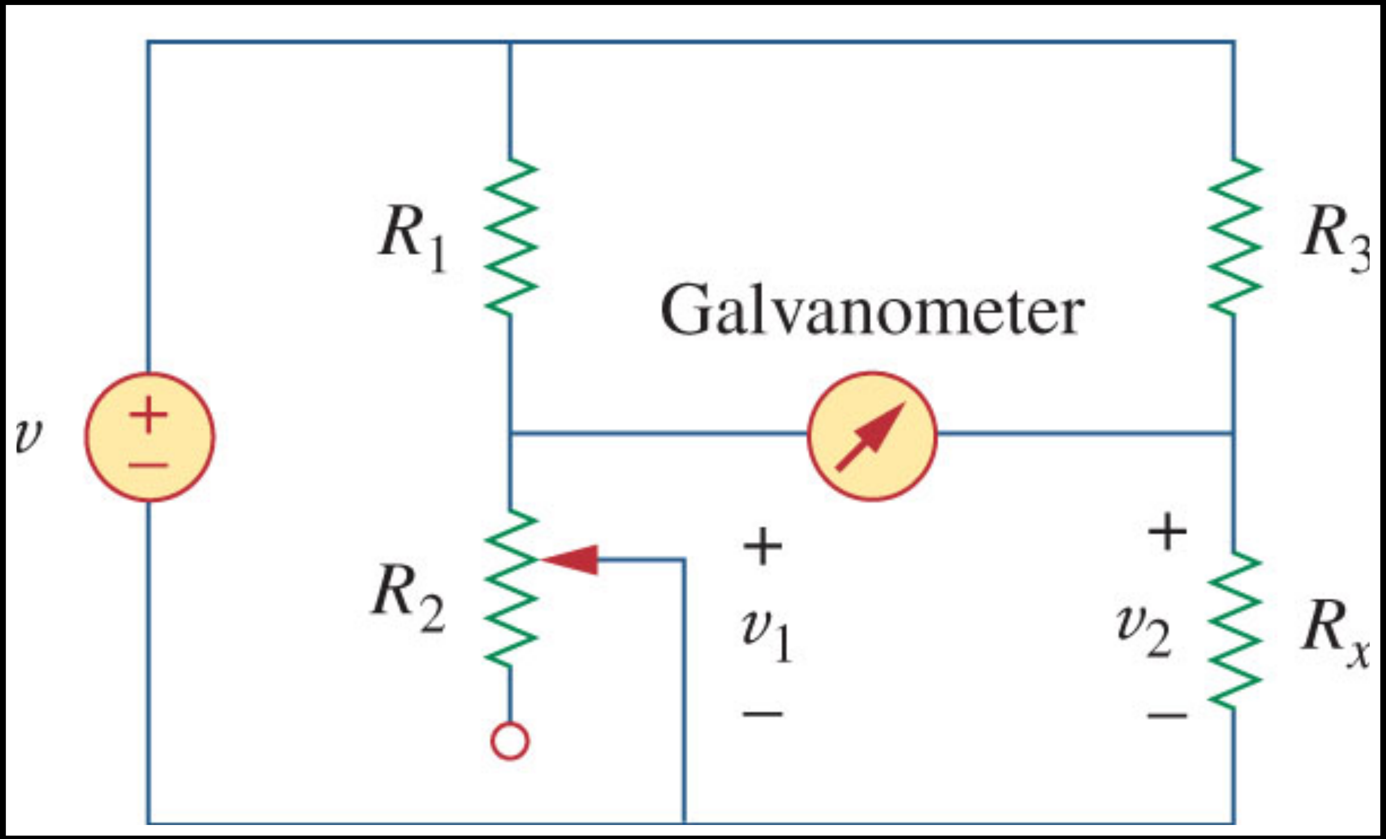
for measuring resistance very accurately





$$R_x = \frac{R_3}{R_1} R_2$$

- **Based on the principle of the voltage divider**
- **Using three known resistors and a galvanometer, an unknown resistor** R_X **can be tested**
- **The variable resistor** R_2 **is adjusted until the galvanometer shows zero current**
- **At this point, the bridge is "balanced" and the voltages from the two dividers are equal**

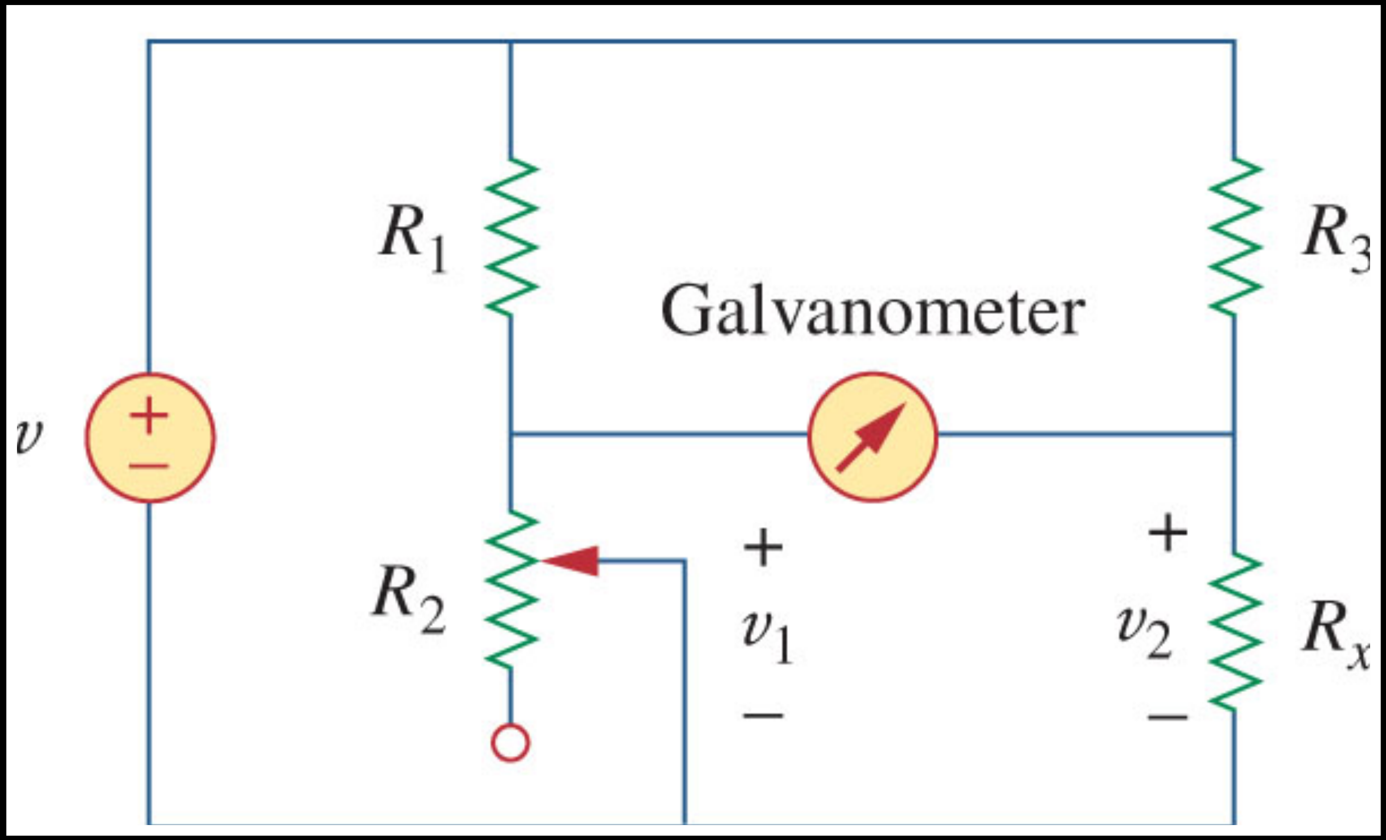


Current through galvo under unbalanced condition:

$$I = \frac{V_{Th}}{R_{Th} + R_m}$$

Simple Example
Wheatstone bridge

- R1 = 500 Ohm and R3 = 200 Ohm and the bridge is balanced when R2 is adjusted to be 125 Ohm. Find Rx



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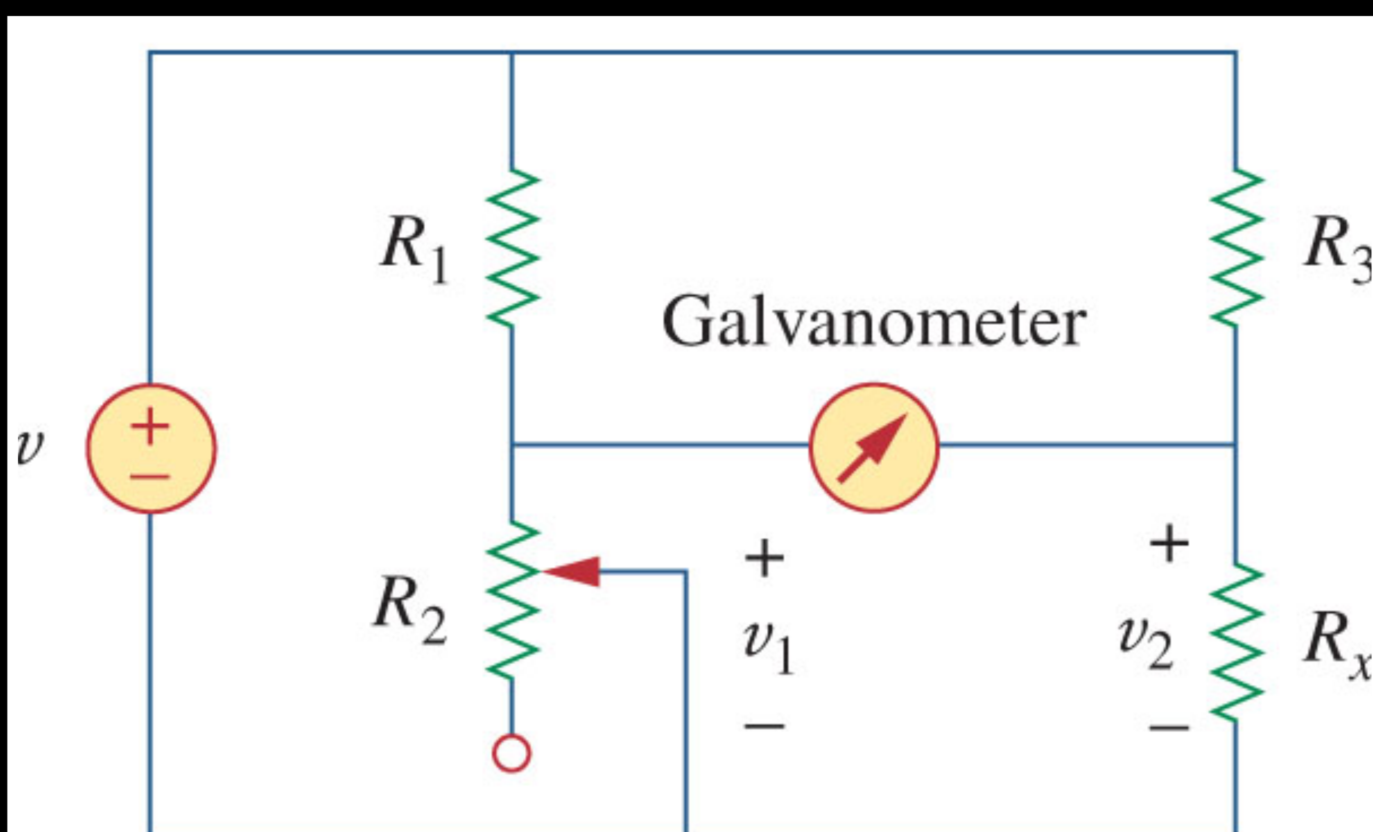


$$R_x = \frac{R_3}{R_1} R_2$$

Rx = (200/500)x125 =

= 50 [Ohm]

Super Simple Example Wheatstone bridge

- A Wheatstone bridge that R1 = R2 = 2 kOhm. R2 is adjusted until no current flows through the galvanometer. At this point, R2 = 6.3 kOhm.

  What is the value of the unknown resistance?

Super Simple Example Wheatstone bridge

- A Wheatstone bridge that R1 = R2 = 2 kOhm. R2 is adjusted until no current flows through the galvanometer. At this point, R2 = 6.3 kOhm.

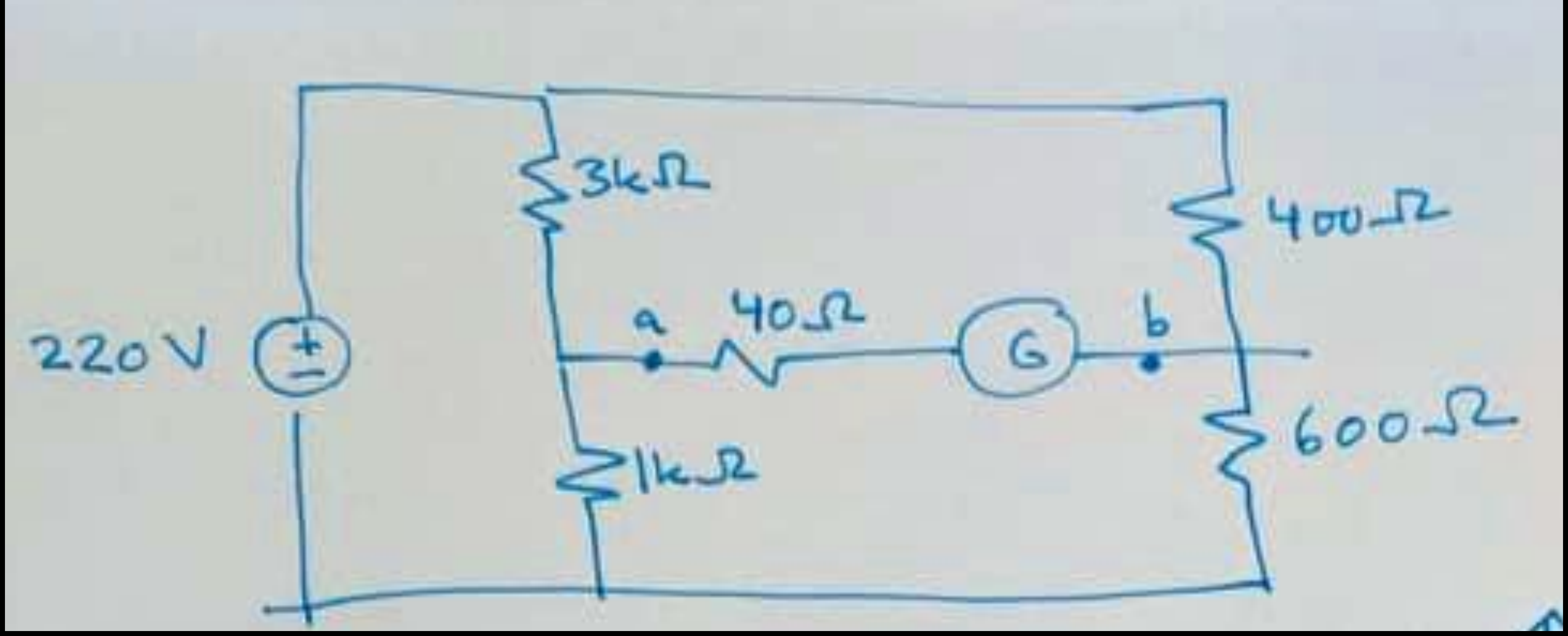
  What is the value of the unknown resistance?

  Answer: Rx = 6.3 kOhm

Example 3
Wheatstone bridge

- Find the current through the galvanometer G:

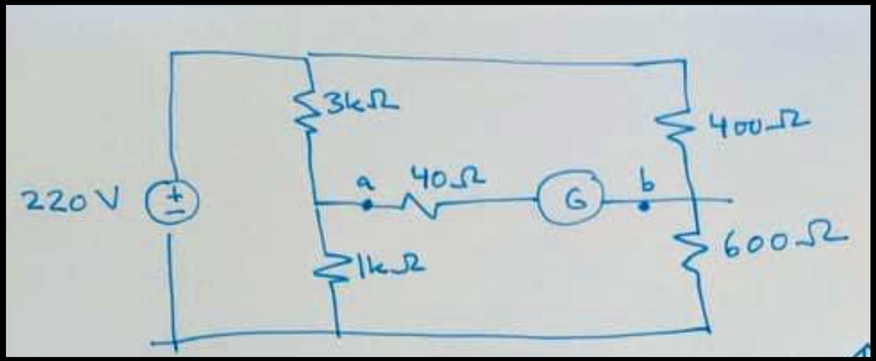
Example of a Wheatstone bridge:

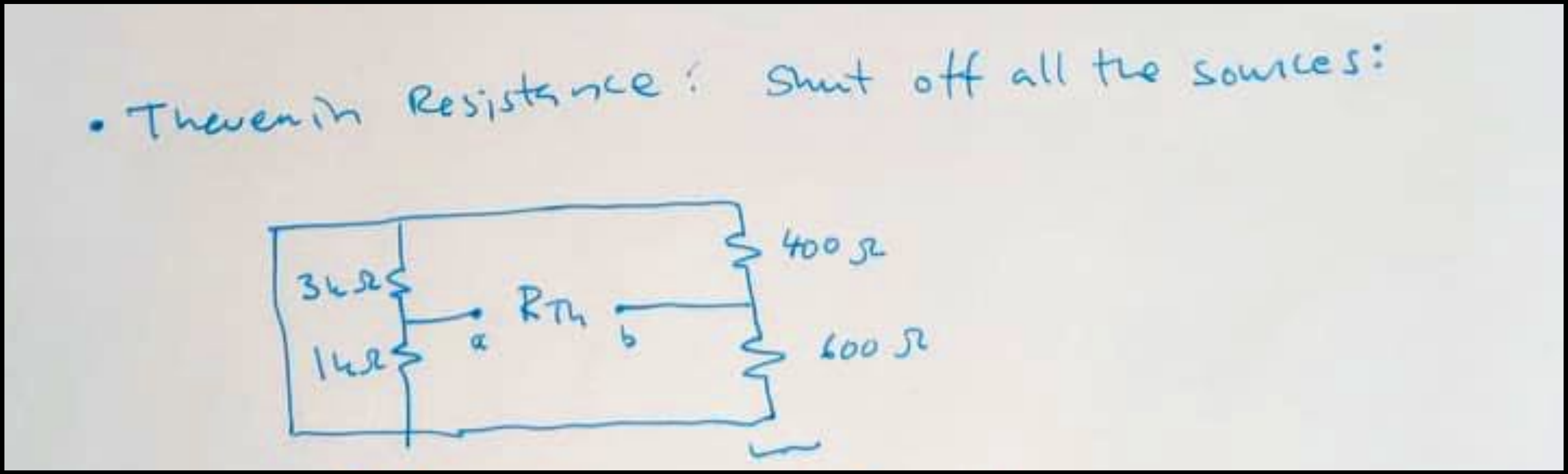


Often drawn like this instead:

Finding R_{Th}

Our bridge:





Finding R_{Th}

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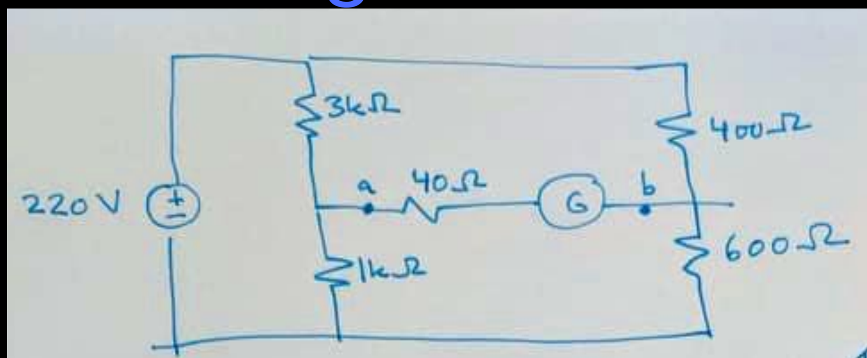
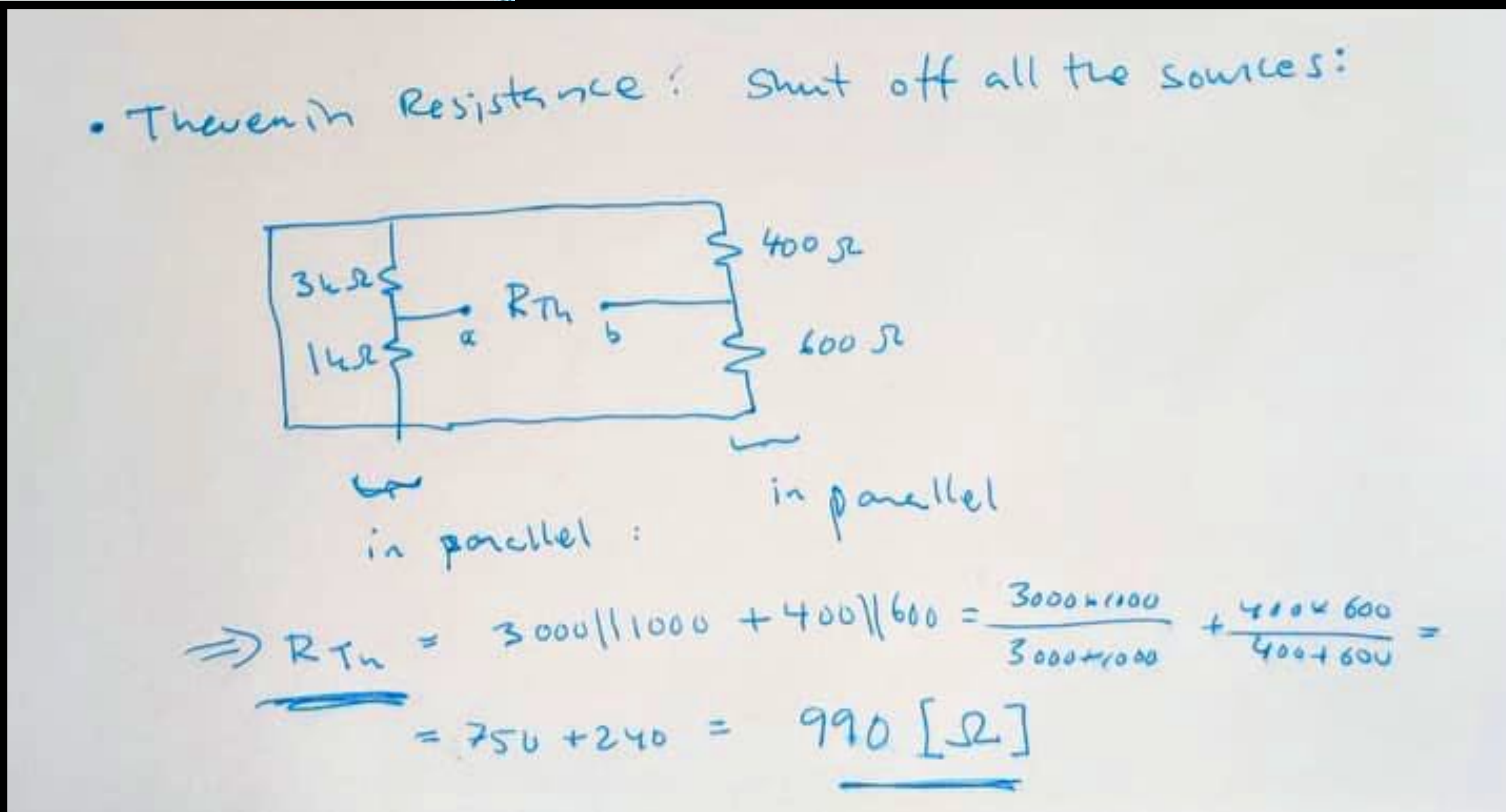
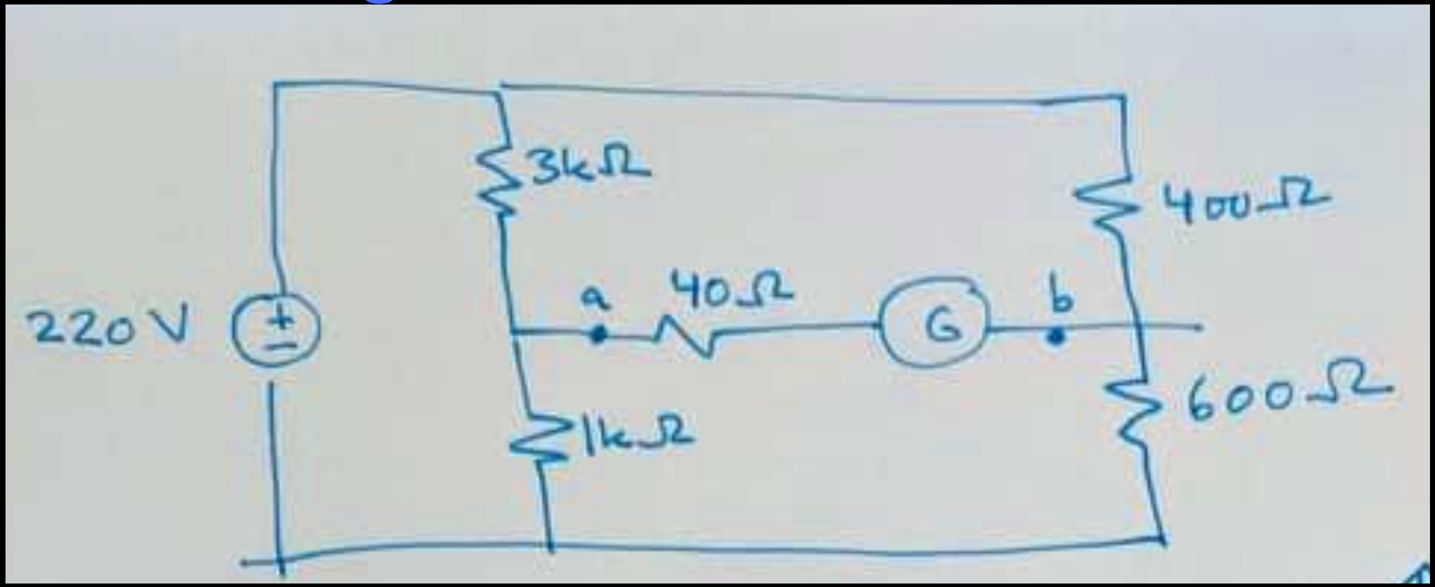
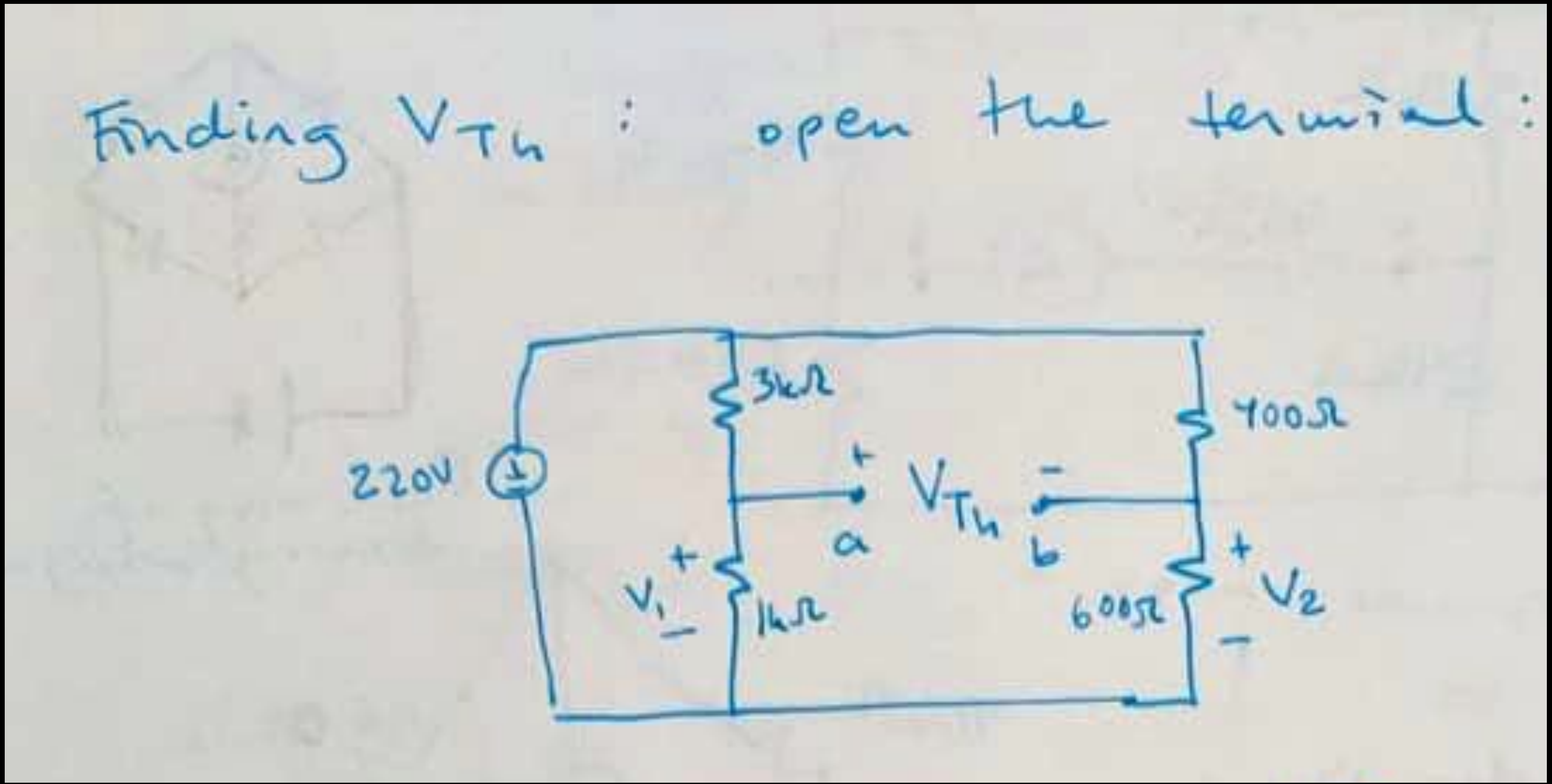


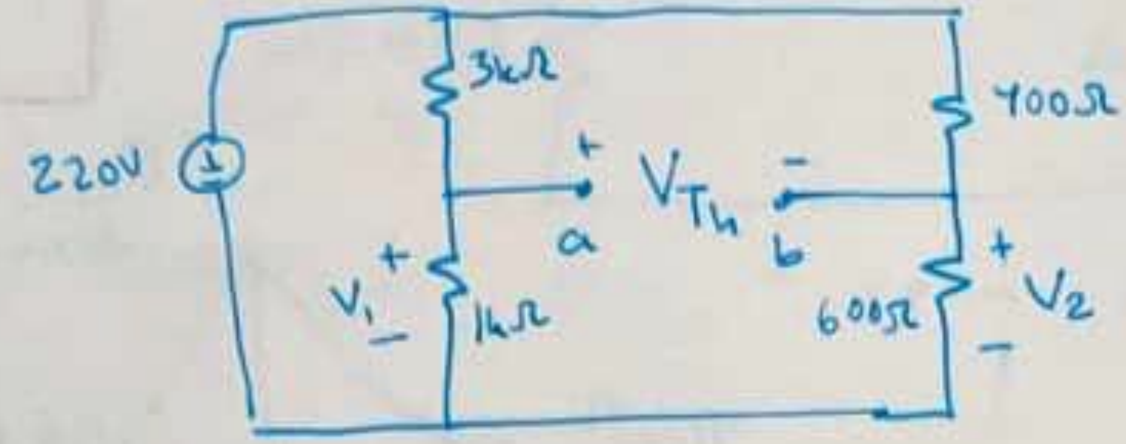


Finding V_{Th}

Our bridge:



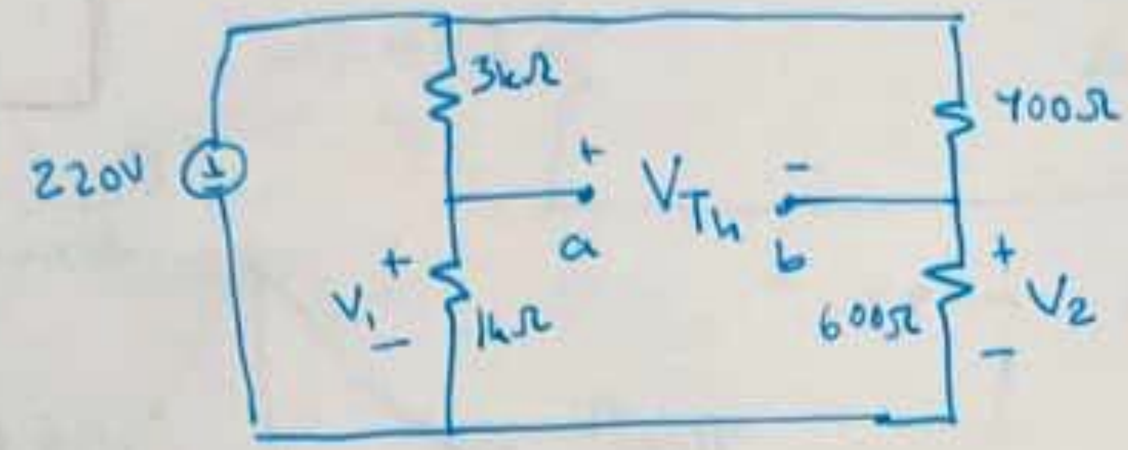




Voltage division :

$$V_1 = \frac{1000}{1000+3000}(220) = 55V$$

$$V_2 = \frac{600}{600+400}(220) = 132V$$



Voltage division:

$$V_1 = \frac{1000}{1000+3000}(220) = 55\text{V}$$

$$V_2 = \frac{600}{600+400}(220) = 132\text{V}$$

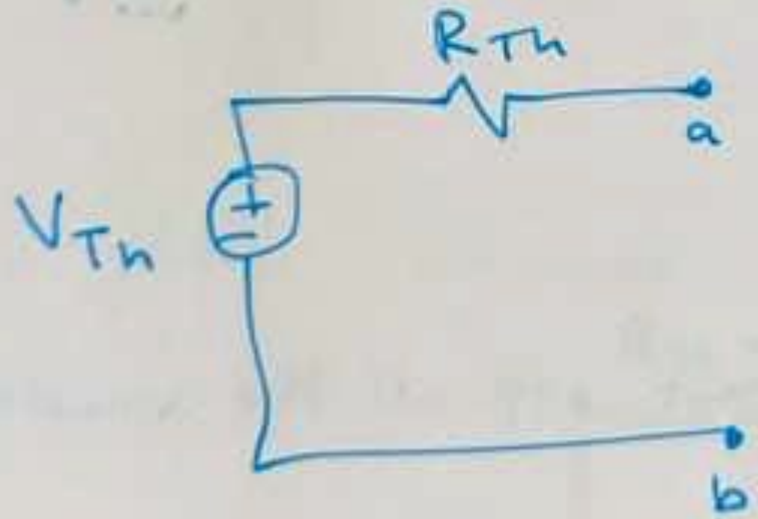
KVL around loop ab:

$$-V_1 + V_{Th} + V_2 = 0 \Rightarrow V_{Th} = V_1 - V_2 = 55 - 132 = -77\text{V}$$

- Now we have found R_{Th} = 990 Ohm and V_{Th} = -77 V

Find current through the galvo:

Draw Thevenin equiv. circuit:



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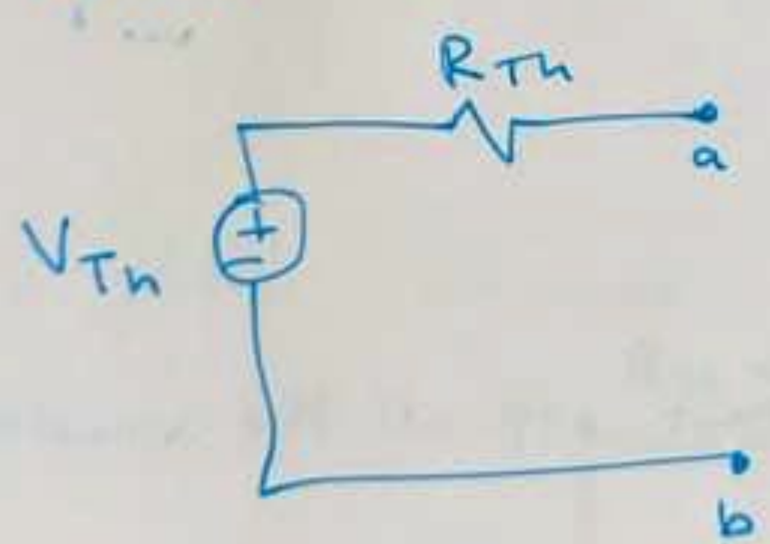
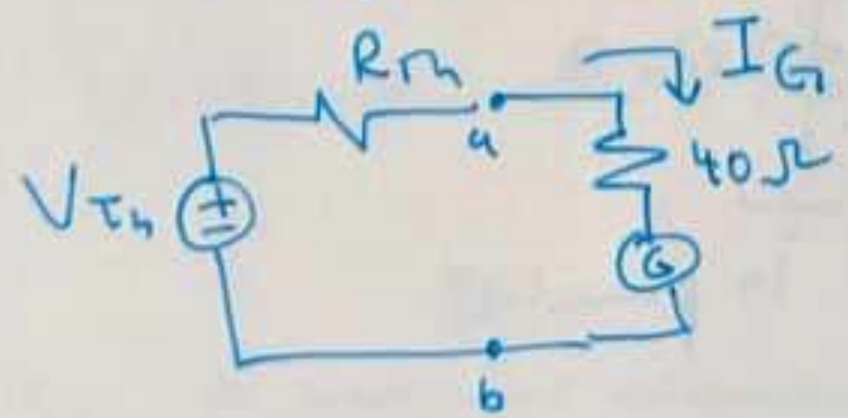


we put on our "Load"

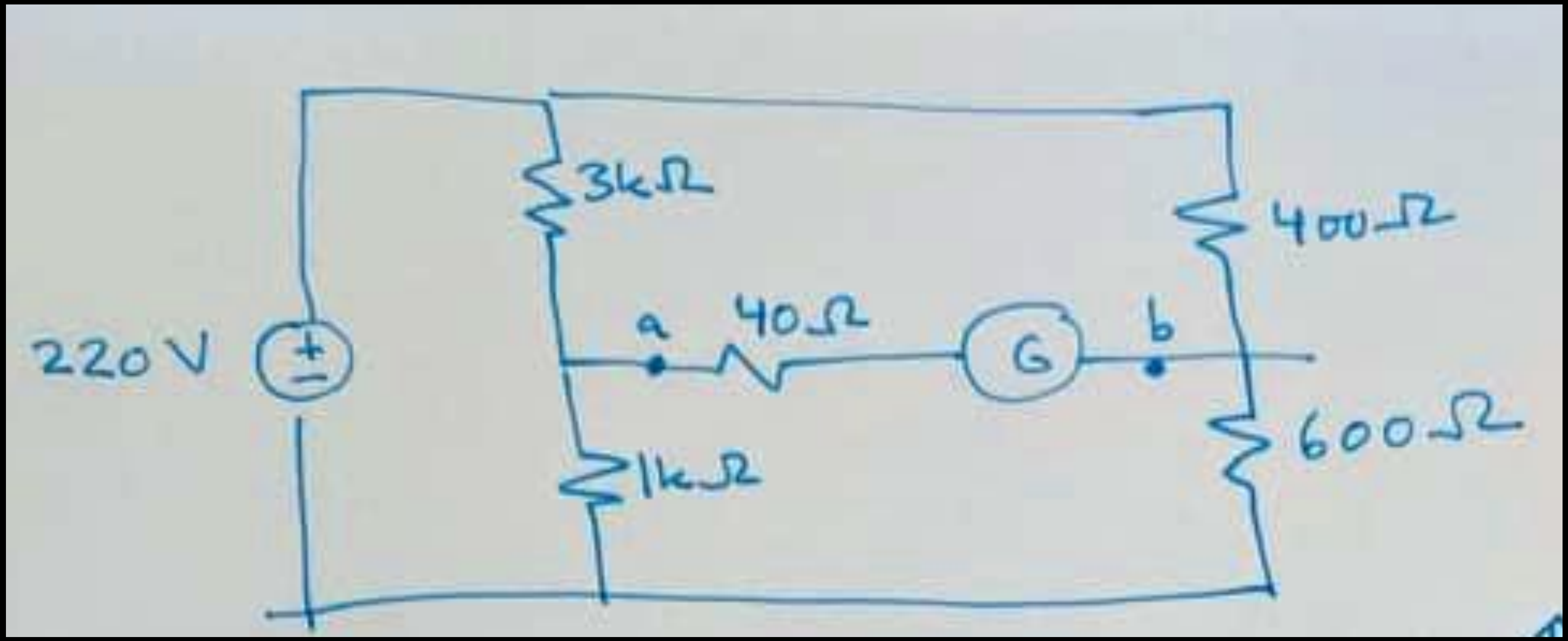


$$I_G = \frac{V_{Th}}{R_{Th}+R_{galvo}} = \frac{-77}{990+40} = -74.6\,[mA]$$

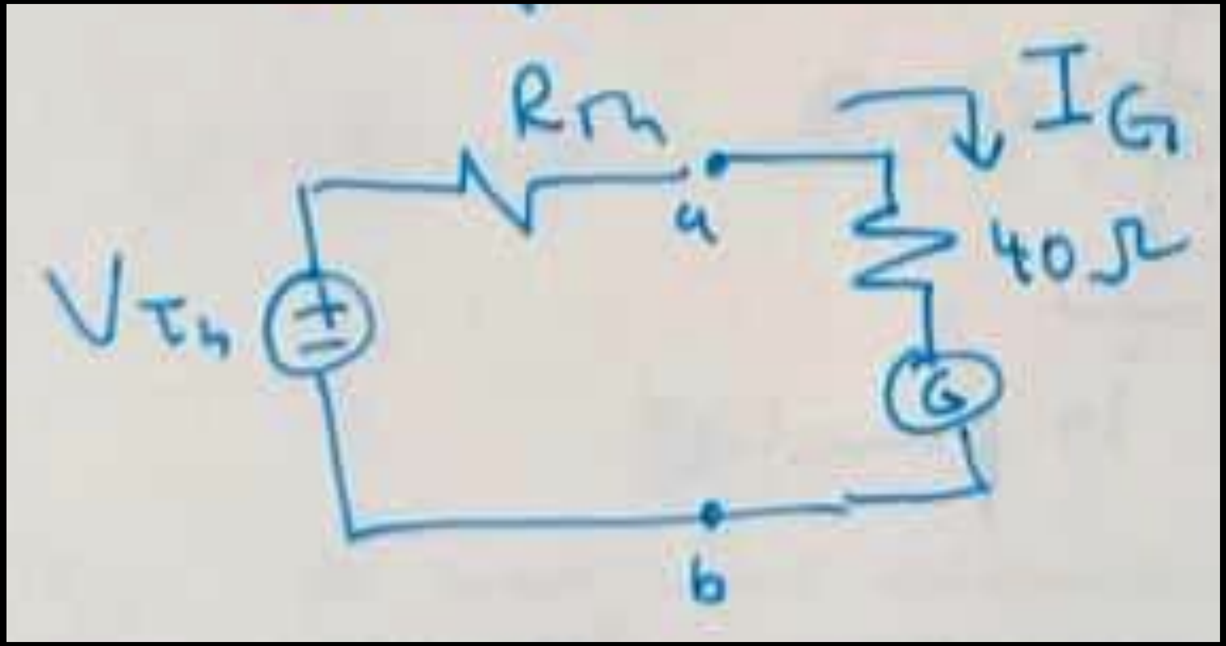
⇒ current flows from b to a

Summary:

Our Wheatstone bridge:

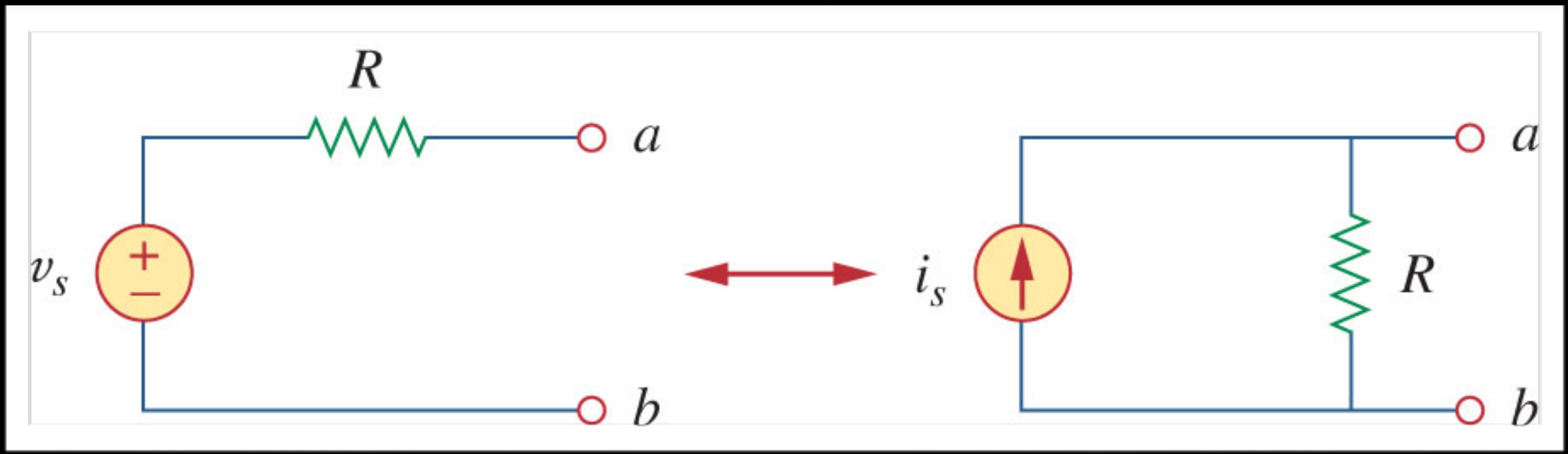


Thevenin equivalent:



Now: Remember Source Transformation

We can replace a voltage source in series with a resistor by a current source in parallel with a resistor (and *vice versa*).



These sources have equivalent behavior at their terminals. If the sources are turned off the resistance at the terminals are both R. If the terminals are short circuited, the currents need to be the same, so that:

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

Thevenin and Norton equivalent circuits

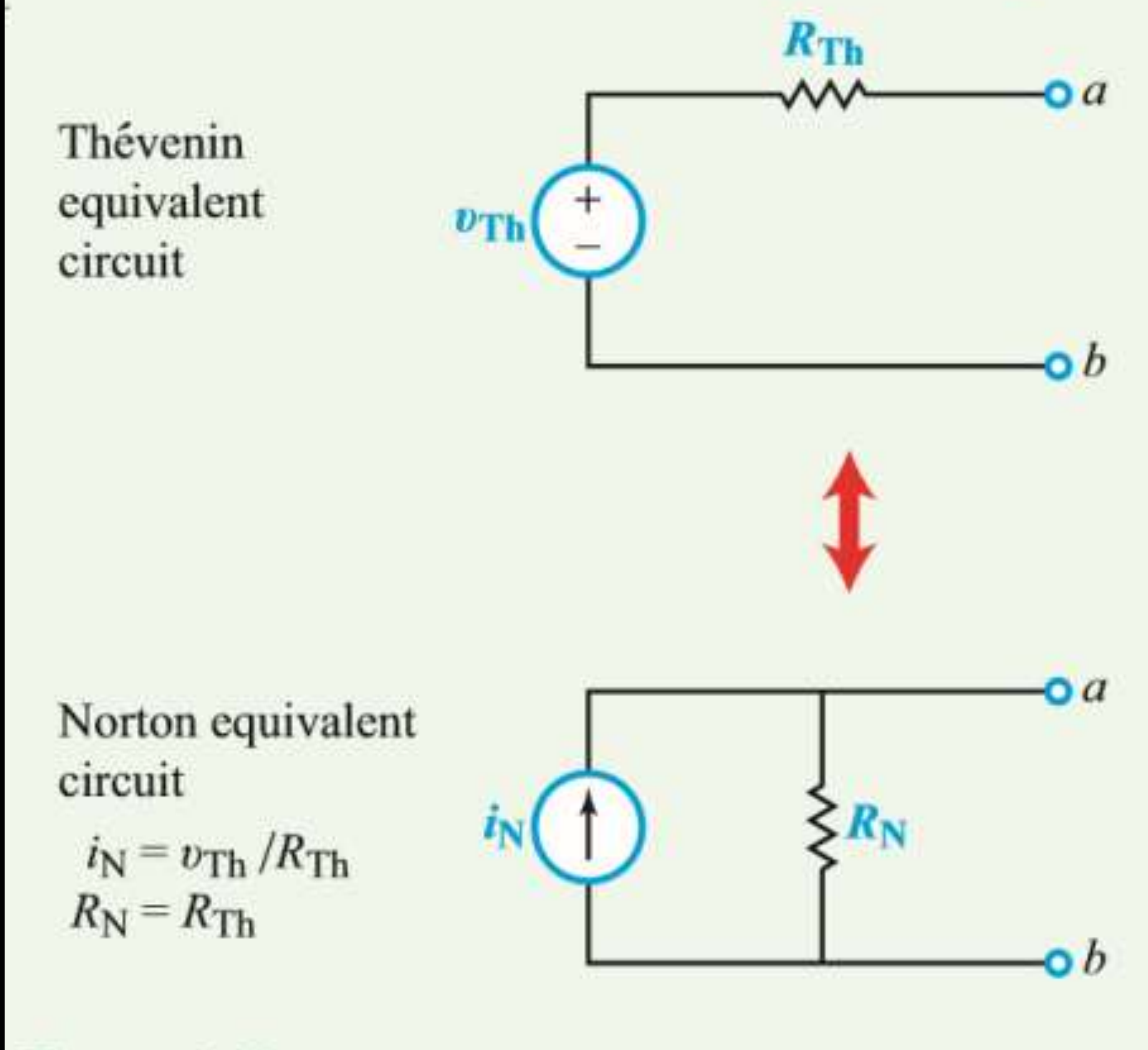
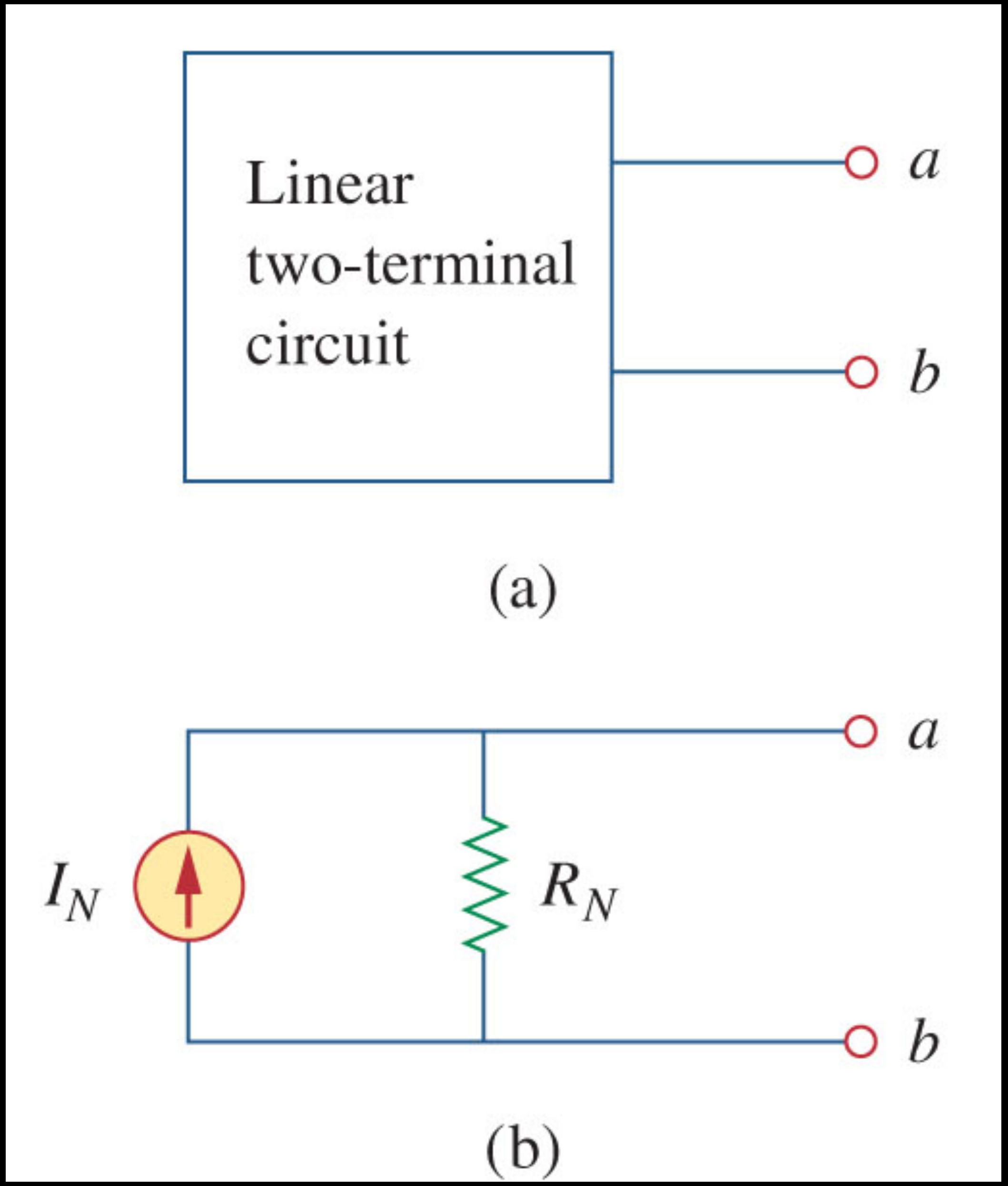


Figure 3-28: Equivalence between Thévenin and Norton equivalent circuits, consistent with the source transformation method of Section 2-3.4.

Norton’s Theorem

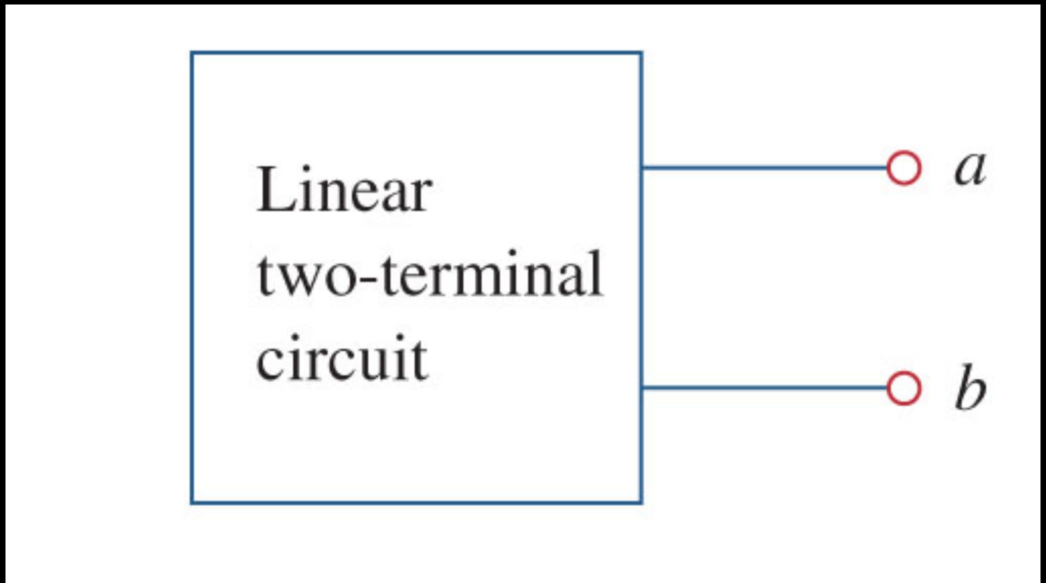
- Similar to Thevenin’s theorem, Norton’s theorem states that a linear two terminal circuit may be replaced with an equivalent circuit containing a **resistor and a current source**
- Norton resistance is the same as Thevenin resistance

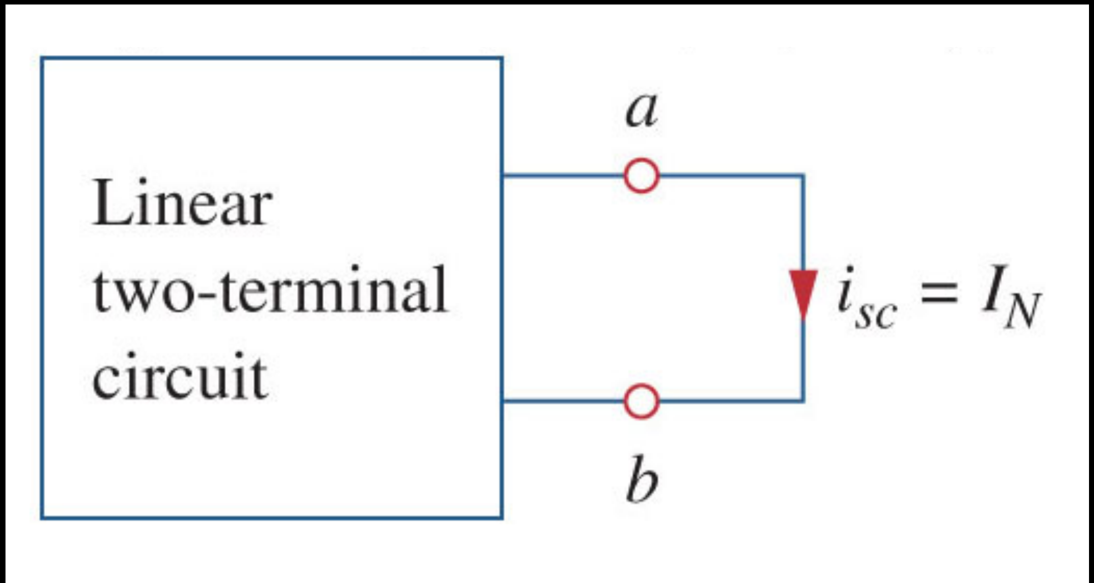


Norton Current I_N

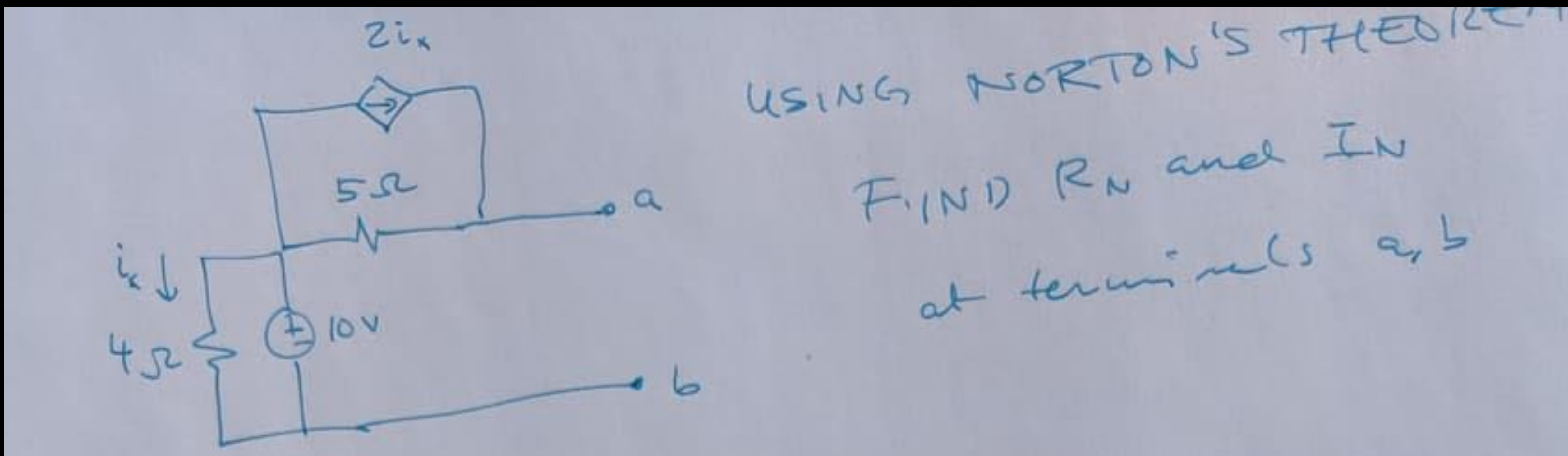
- The Norton current I_N is found by short circuiting the circuit's terminals and measuring the resulting current:

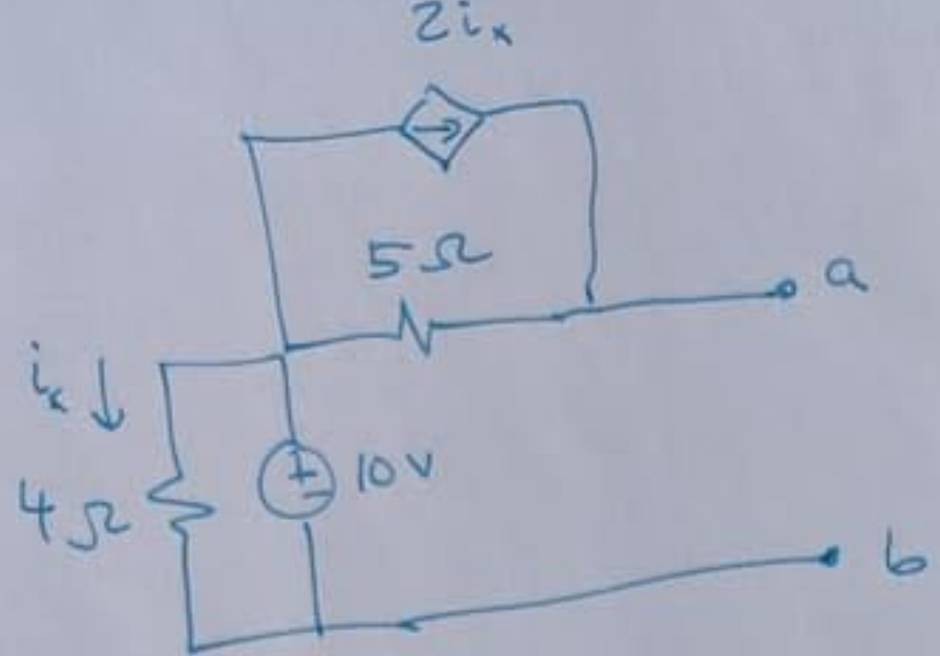
$$I_N = i_{SC}$$





Example: Norton





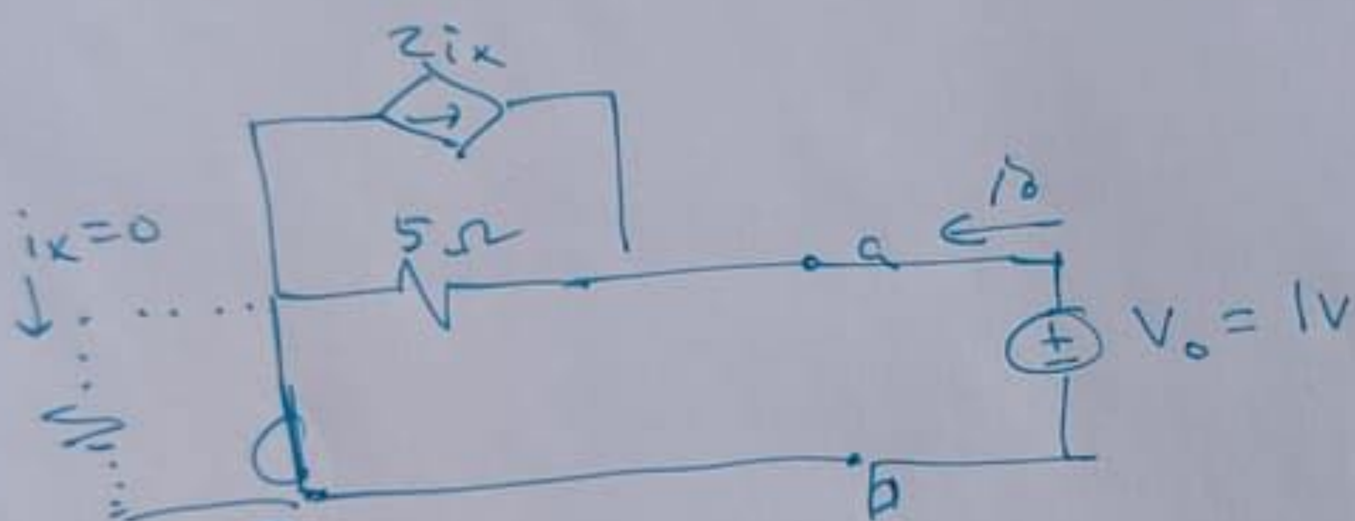
USING NORTON'S THEOREM

FIND R_N and I_N

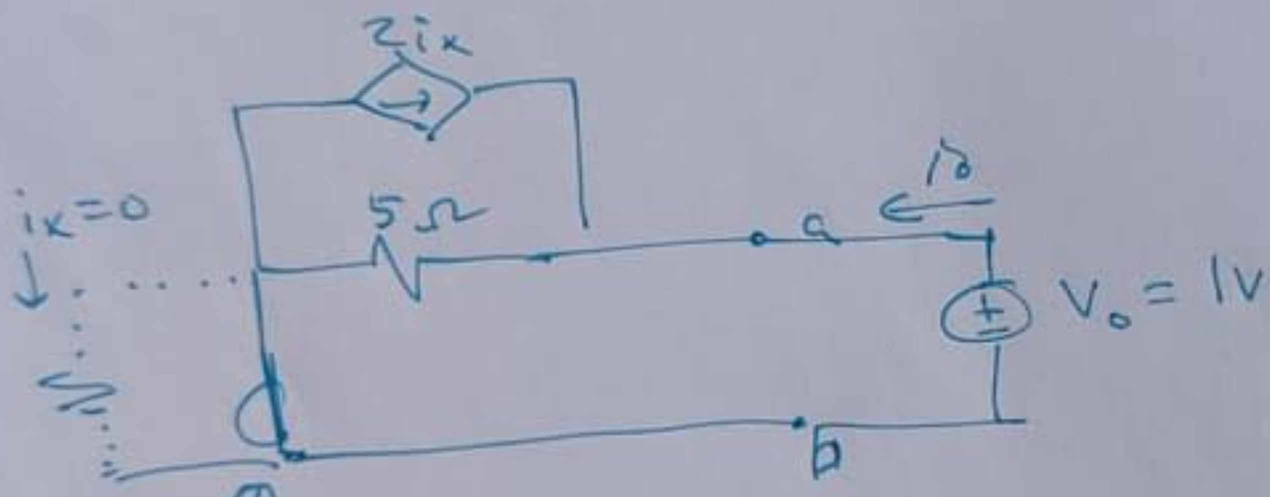
at terminals a, b

- Set independent voltage source to zero
- We have dependent source ⇒ connect voltage source V_o to ab terminal.

  (we choose $V_o = 1V$)



4Ω is shorted out



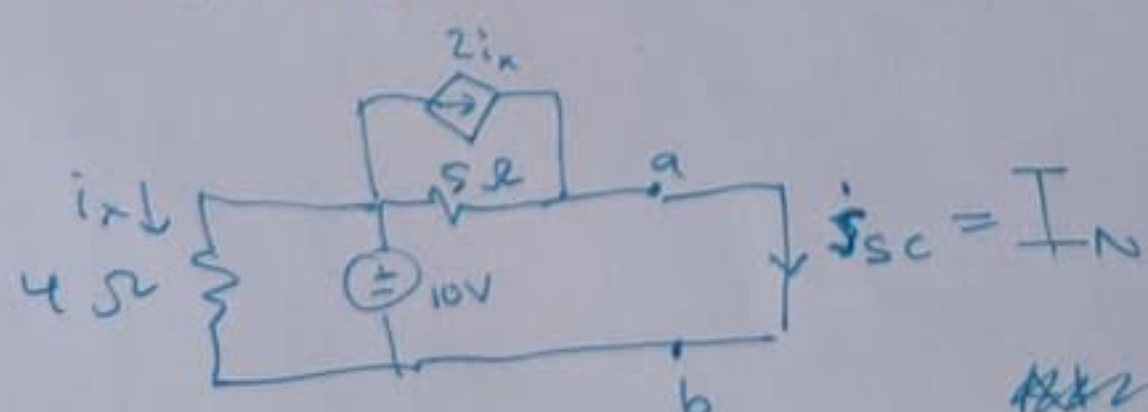
4Ω is shorted out

Everything is now in parallel

At node a: $\quad i_0 = \dfrac{1V}{5\Omega} = 0.2\ A$

we have $\quad R_N = \dfrac{V_0}{i_0} = \dfrac{1}{0.2} = 5\ [\Omega]$

Find I_N: short terminal ab and find i_{sc}



All still in parallel:

$i_x = \dfrac{10}{4} = 2.5\ (A]$

At node a, KCL gives: $\quad i_{sc} = \dfrac{10}{5} + 2i_x = 2 + 2(2.5) = \underline{\underline{7A}}$

Norton and Thevenin Summary:

With V_{Th}, I_N, and $R_{Th} = R_N$ related, finding the Thevenin or Norton equivalent circuit requires that we find:

- The open-circuit voltage across terminals *a and b.*
- The short-circuit current at terminals *a and b.*
- The equivalent or input resistance at terminals *a and b when* all independent sources are turned off.

Norton *versus* Thevenin

Norton current and Thevenin voltage are related to each other (source substitution) as:

$$I_N = V_{Th} / R_{Th}$$

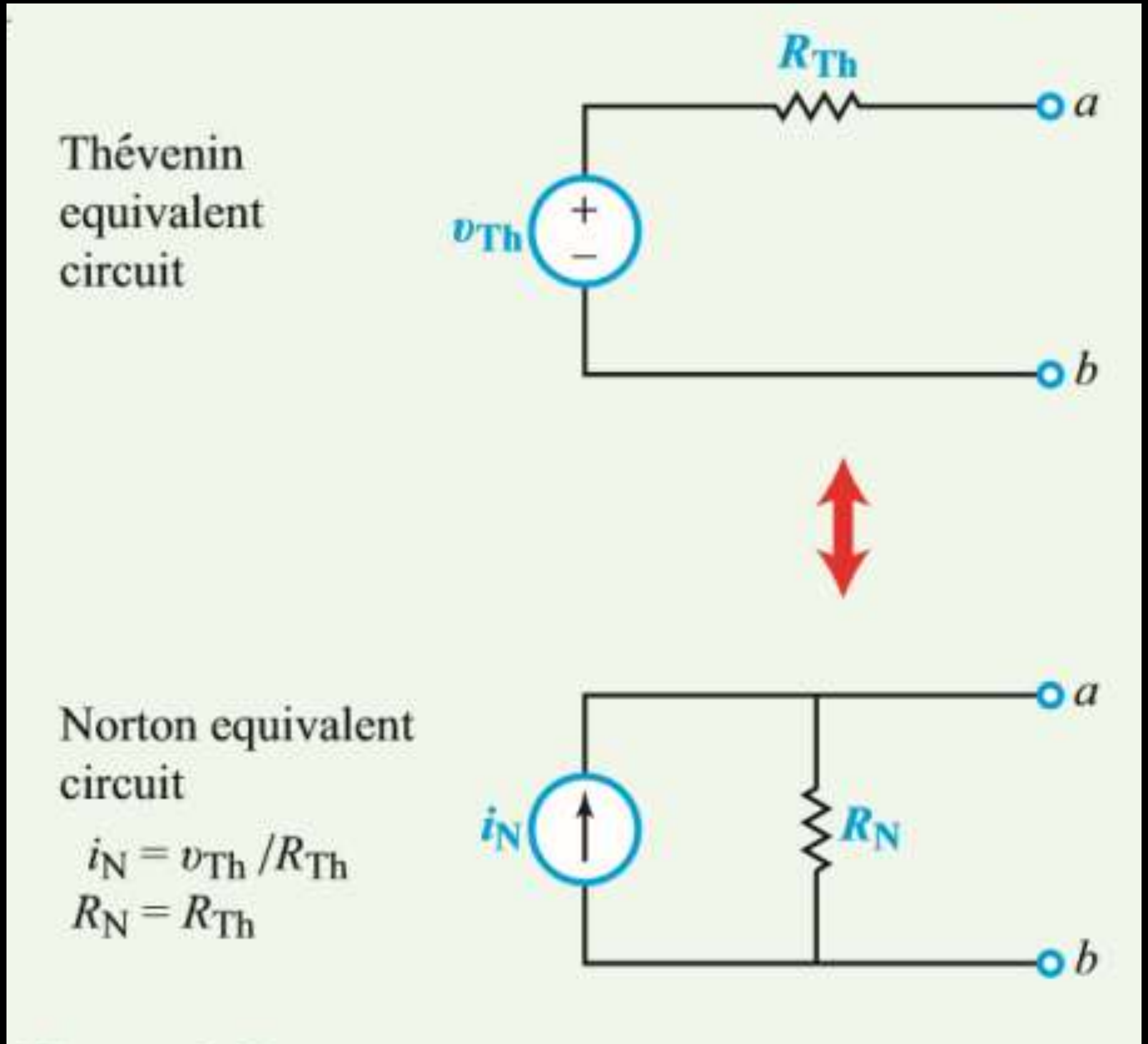


Figure 3-28: Equivalence between Thévenin and Norton equivalent circuits, consistent with the source transformation method of Section 2-3.4.

QUIZ TIME!!!