

ECE101 F19 Lecture 8, Nov. 26 2019

HW #9 for Quiz 9 on Dec. 3

- | | |
|---------------|-----------------|
| 1) Prob. 6.32 | 10) Prob 7.10 |
| 2) 6.38 | 11) 7.20 |
| 3) 6.48 | 12) 7.29 |
| 4) 6.52 | 13) 7.32 |
| 5) 6.54 | 14) 7.40 |
| 6) 6.58 | |
| 7) prob 7.1 | Quiz 8 Avg 6.07 |
| 8) 7.3 | Quiz 8 Avg 1.95 |
| 9) 7.6 | Quiz 8 Avg 2.18 |

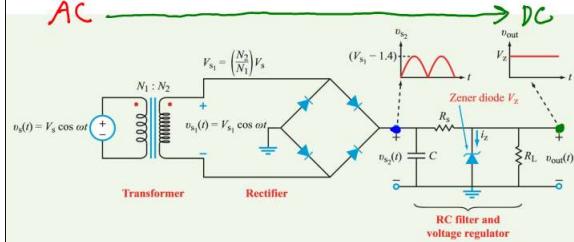


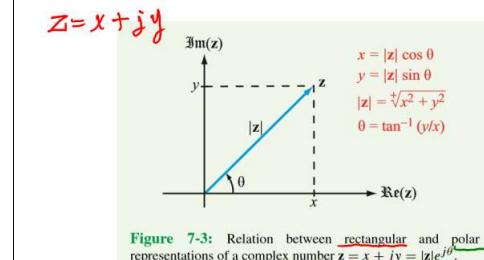
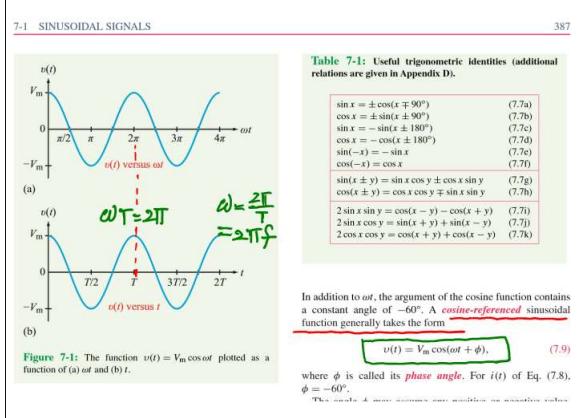
Figure 7-40: Complete power-supply circuit.

Phasor Method

$$V_C = V_S \frac{Z_C}{R + Z_C} = 1 \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$= \frac{1}{\sqrt{1 + (\omega RC)^2}} e^{j\arctan(-\omega RC)} = \frac{1}{\sqrt{1 + (\omega RC)^2}} e^{-j\theta}$$

$$\Rightarrow V_C(t) = \frac{1}{\sqrt{1 + (\omega RC)^2}} \cos(\omega t - \theta)$$



which leads to the relations

$$x = |z| \cos \theta, \quad y = |z| \sin \theta, \quad (7.17)$$

$$|z| = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x). \quad (7.18)$$

$$\begin{aligned}
 v_s(t) &= V_0 \cos(\omega t + \phi) \\
 &= \Re\{ V_0 e^{j(\omega t + \phi)} \} \\
 &= \Re\{ \underbrace{V_0 e^{j\phi}}_{\text{phasor}} e^{j\omega t} \} \\
 &= \Re\{ \underbrace{V_s}_{\text{phasor}} e^{j\omega t} \}
 \end{aligned}$$

$$\begin{aligned}
 v_s(t) &= \sqrt{V_0} \sin(\omega t + \phi) \\
 &= V_0 \cos(\omega t + \phi - \frac{\pi}{2}) \\
 &= \Re\{ \underbrace{V_0 e^{j(\phi - \frac{\pi}{2})}}_{\text{phasor}} e^{j\omega t} \} \\
 &= \Re\{ \underbrace{V_s}_{\text{phasor}} e^{j\omega t} \}
 \end{aligned}$$

If $\phi = 0$, $V_s = V_0 e^{-j\pi/2}$
 $= V_0 \cos \frac{\pi}{2} - j V_0 \sin \frac{\pi}{2}$
 $= -j V_0$ ($= V_0 e^{-j\pi/2}$)

Any cosinusoidally **time-varying function** $x(t)$, representing a voltage or a current, can be expressed in the form

$$x(t) = \Re[\underbrace{\mathbf{X}}_{\text{phasor}} e^{j\omega t}], \quad (7.28)$$

where \mathbf{X} is a **time-independent** function called the **phasor counterpart** of $x(t)$. Thus, $x(t)$ is defined in the time domain, while its counterpart \mathbf{X} is defined in the phasor domain.

- To distinguish phasor quantities from their time-domain counterparts, phasors are always represented by **bold letters** in this book. ◀

Time Domain $v(t) = V_0 \cos \omega t$	Phasor Domain $\mathbf{V} = V_0 \underline{e^{j\phi}}$ (7.31a)
$v(t) = V_0 \cos(\omega t + \phi)$	$\mathbf{V} = V_0 e^{j\phi}$. (7.31b)

If $\phi = -\pi/2$,

$$v(t) = V_0 \cos(\omega t - \pi/2) \leftrightarrow \mathbf{V} = V_0 e^{-j\pi/2}. \quad (7.32)$$

Since $\cos(\omega t - \pi/2) = \cos(\pi/2 - \omega t) = \sin \omega t$ and

$$e^{-j\pi/2} = \cos(\pi/2) - j \sin(\pi/2) = -j,$$

Eq. (7.32) reduces to

$$v(t) = V_0 \sin \omega t \leftrightarrow \mathbf{V} = -j V_0, \quad (7.33)$$

which can be generalized to

$$v(t) = V_0 \sin(\omega t + \phi) \leftrightarrow \mathbf{V} = V_0 e^{j(\phi - \pi/2)}. \quad (7.34)$$

$$i(t) = \Re[\mathbf{I} e^{j\omega t}], \quad (7.35)$$

where \mathbf{I} may be complex but, by definition, not a function of time. The derivative di/dt is given by

$$\frac{di}{dt} = \frac{d}{dt} [\Re(\mathbf{I} e^{j\omega t})] = \Re \left[\frac{d}{dt} (\mathbf{I} e^{j\omega t}) \right] = \Re \left[\underbrace{j\omega \mathbf{I}}_{\text{phasor of } di/dt} e^{j\omega t} \right], \quad (7.36)$$

$$\frac{di}{dt} \leftrightarrow j\omega \mathbf{I}, \quad (7.37)$$

or:

- Differentiation of a time function $i(t)$ in the time domain is equivalent to multiplication of its phasor counterpart \mathbf{I} by $j\omega$ in the phasor domain. ◀

Similarly,

$$\begin{aligned}
 \int i dt &= \int \Re[\mathbf{I} e^{j\omega t}] dt \\
 &= \Re \left[\int \mathbf{I} e^{j\omega t} dt \right] = \Re \left[\underbrace{\frac{1}{j\omega}}_{\text{phasor of } \int i dt} e^{j\omega t} \right].
 \end{aligned} \quad (7.38)$$

or

$$\int i dt \leftrightarrow \frac{\mathbf{I}}{j\omega}, \quad (7.39)$$

Table 7-3: Time-domain sinusoidal functions $x(t)$ and their cosine-reference phasor-domain counterparts \mathbf{X} , where $x(t) = \Re[\mathbf{X}e^{j\omega t}]$.

$x(t)$	\mathbf{X}
$A \cos \omega t$	$\leftrightarrow A$
$A \cos(\omega t + \phi)$	$\leftrightarrow Ae^{j\phi}$
$-A \cos(\omega t + \phi)$	$\leftrightarrow Ae^{j(\phi \pm \pi)}$
$A \sin \omega t$	$\leftrightarrow Ae^{-j\pi/2} = -jA$
$A \sin(\omega t + \phi)$	$\leftrightarrow Ae^{j(\phi - \pi/2)}$
$-A \sin(\omega t + \phi)$	$\leftrightarrow Ae^{j(\phi + \pi/2)}$
$\frac{d}{dt}(x(t))$	$\leftrightarrow j\omega \mathbf{X}$
$\frac{d}{dt}[A \cos(\omega t + \phi)]$	$\leftrightarrow j\omega Ae^{j\phi}$
$\int x(t) dt$	$\leftrightarrow \frac{1}{j\omega} \mathbf{X}$
$\int A \cos(\omega t + \phi) dt$	$\leftrightarrow \frac{1}{j\omega} Ae^{j\phi}$

$$\begin{aligned} & -A \cos(\omega t + \phi) \\ & \overline{A} \cos(\omega t + \phi) \\ & \times e^{\pm j\pi} \\ & = \Re\{A e^{j(\phi \pm \pi)}\} \\ & \Downarrow \\ & A e^{j(\phi \pm \pi)} \end{aligned}$$

$x(t)$	\mathbf{X}
$A \cos \omega t$	$\leftrightarrow A$
$A \cos(\omega t + \phi)$	$\leftrightarrow Ae^{j\phi}$
$-A \cos(\omega t + \phi)$	$\leftrightarrow Ae^{j(\phi \pm \pi)}$
$A \sin \omega t$	$\leftrightarrow Ae^{-j\pi/2} = -jA$
$A \sin(\omega t + \phi)$	$\leftrightarrow Ae^{j(\phi - \pi/2)}$
$-A \sin(\omega t + \phi)$	$\leftrightarrow Ae^{j(\phi + \pi/2)}$
$\frac{d}{dt}(x(t))$	$\leftrightarrow j\omega \mathbf{X}$
$\frac{d}{dt}[A \cos(\omega t + \phi)]$	$\leftrightarrow j\omega Ae^{j\phi}$
$\int x(t) dt$	$\leftrightarrow \frac{1}{j\omega} \mathbf{X}$
$\int A \cos(\omega t + \phi) dt$	$\leftrightarrow \frac{1}{j\omega} Ae^{j\phi}$

► The **impedance** \mathbf{Z} of a circuit element is defined as the ratio of the phasor voltage across it to the phasor current entering through its plus (+) terminal, ◀

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad (\Omega), \quad (7.45)$$

and the unit of \mathbf{Z} is the ohm (Ω). For a resistor, Eq. (7.44) gives

$$\mathbf{Z}_R = \frac{\mathbf{V}_R}{\mathbf{I}_R} = R. \quad (7.46)$$

Phasors \mathbf{V}_L and \mathbf{I}_L are related to their time-domain counterparts by

$$\text{and } \mathbf{V}_L = \Re[\mathbf{V}_L e^{j\omega t}] \quad (7.48a)$$

$$i_L = \Re[\mathbf{I}_L e^{j\omega t}]. \quad (7.48b)$$

Consequently,

$$\Re[\mathbf{V}_L e^{j\omega t}] = L \frac{d}{dt} [\Re(\mathbf{I}_L e^{j\omega t})] = \Re[j\omega L \mathbf{I}_L e^{j\omega t}], \quad (7.49)$$

which leads to

$$\mathbf{V}_L = j\omega L \mathbf{I}_L. \quad (7.50)$$

Hence, the impedance of an inductor L is

$$\mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}_L} = j\omega L. \quad (7.51)$$

Table 7-4: Summary of $v-i$ properties for R , L , and C .

Property	R	L	C
$v-i$	$v = Ri$	$v = L \frac{di}{dt}$	$i = C \frac{dv}{dt}$
$\mathbf{V}-\mathbf{I}$	$\mathbf{V} = RI$	$\mathbf{V} = j\omega L \mathbf{I}$	$\mathbf{V} = \frac{1}{j\omega C}$
\mathbf{Z}	R	$j\omega L$	$\frac{1}{j\omega C}$
dc equivalent	R	Short circuit	Open circuit
High-frequency equivalent	R	Open circuit	Short circuit
Frequency response	$ Z_R $	$ Z_L $	$ Z_C $

Capacitors

Since for a capacitor

$$\text{and } i_C = C \frac{dv_C}{dt}, \quad (7.52)$$

it follows that in the phasor domain,

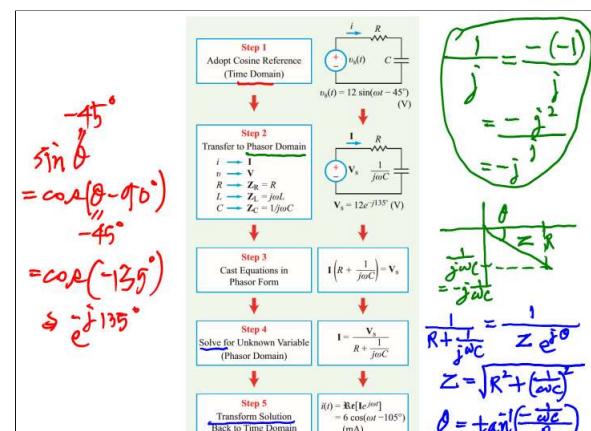
$$\mathbf{i}_C = j\omega C \mathbf{V}_C \quad (7.53)$$

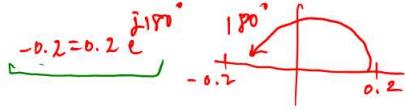
and the impedance of a capacitor C is

$$\mathbf{Z}_C = \frac{\mathbf{V}_C}{\mathbf{I}_C} = \frac{1}{j\omega C}. \quad (7.54)$$

Because \mathbf{Z}_L and \mathbf{Z}_C are, respectively, directly and inversely proportional to ω , \mathbf{Z}_L and \mathbf{Z}_C assume inverse roles as ω approaches zero and ∞ .

► In the phasor domain, a capacitor behaves like an open circuit at dc and like a short circuit at very high frequencies. ◀





Prob. 7.2 Express the current waveform

$$i(t) = 0.2 \cos(\omega t + 60^\circ) \text{ mA}$$

in standard cosine form and then determine the following:

(a) Its amplitude, frequency, and phase angle.

(b) $i(t)$ at $t = 0.1 \text{ ns}$.

$$\begin{aligned} & 0.2 e^{j180^\circ} \operatorname{Re} \{ e^{j(\omega t + 60^\circ)} \} \\ &= \operatorname{Re} \{ 0.2 e^{j180^\circ} e^{j(\omega t + 60^\circ)} \} \\ &= \operatorname{Re} \{ 0.2 e^{j240^\circ} e^{j\omega t} \} \end{aligned}$$

$$\begin{aligned} & (j) \operatorname{Re} \{ V_0 e^{j(\omega t + \phi)} \} \\ &= \operatorname{Re} \{ V_0 e^{j\frac{\pi}{2}} e^{j\omega t} e^{j\phi} \} \\ &= \operatorname{Re} \{ V_0 e^{j(\phi + \frac{\pi}{2})} e^{j\omega t} \} \\ & \quad \text{phasor} \\ &= V_0 \cos(\omega t + \phi + \frac{\pi}{2}) \\ &= V_0 \left[\cos(\omega t + \phi) \cos \frac{\pi}{2} - \sin(\omega t + \phi) \sin \frac{\pi}{2} \right] \\ &= -V_0 \sin(\omega t + \phi) \end{aligned}$$



7.2 Express the current waveform

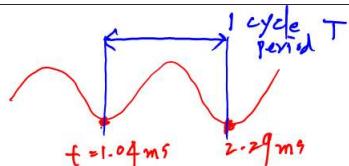
$$i(t) = -0.2 \cos(\underline{\omega t} + 60^\circ) \text{ mA}$$

in standard cosine form and then determine the following:

(a) Its amplitude, frequency, and phase angle.

(b) $i(t)$ at $t = 0.1 \text{ ns}$.

$$\begin{aligned} 6\pi \times 10^9 t &= \omega t = 2\pi(3 \times 10^9) t \\ i(t) &= \operatorname{Re} \{ 0.2 e^{\pm j180^\circ} e^{j(\omega t + 60^\circ)} \} \\ &= \operatorname{Re} \{ 0.2 e^{j(60^\circ \pm 180^\circ)} e^{j\omega t} \} \Rightarrow I = 0.2 e^{-j120^\circ} \end{aligned}$$



7.7 Provide an expression for a 24 V signal that exhibits adjacent minima at $t = 1.04 \text{ ms}$ and $t = 2.29 \text{ ms}$.

$$V = 24 \cos(\omega t + \phi)$$

$$V(t) = 24 \cos(\omega t + \phi)$$

7.7 Provide an expression for a 24 V signal that exhibits adjacent minima at $t = 1.04 \text{ ms}$ and $t = 2.29 \text{ ms}$.

$$\begin{aligned} T &= (2.29 - 1.04) \times 10^{-3} = 1.25 \times 10^{-3} \\ \omega T &= 2\pi = \omega(1.25 \times 10^{-3}) \Rightarrow \omega = \frac{2\pi}{1.25 \times 10^{-3}} \\ \omega &= \frac{2\pi}{1.25 \times 10^{-3}} = 1600\pi \end{aligned}$$

$$V(t) = 24 \cos(1600\pi t + \phi)$$

7.10 Express the following complex numbers in polar form:

(a) $z_1 = 3 + j4$

(b) $z_2 = -6 + j8$

*(c) $z_3 = -6 - j4$

(d) $z_4 = j2$

*(e) $z_5 = (2 + j)^2$

(f) $z_6 = (3 - j2)^3$

(g) $z_7 = (-1 + j)^{1/2}$

$(-1 + j)^{\frac{1}{2}} = (\sqrt{2} e^{j135^\circ})^{\frac{1}{2}}$

$= \sqrt{1.414} e^{j67.5^\circ}$

$0.45 + j1.1$

$1.19 = \sqrt{1.414} (\cos 67.5^\circ + j \sin 67.5^\circ)$

$$\sin \alpha = \cos(\theta - 90^\circ)$$

7.33 Find $i_a(t)$ in the circuit of Fig. P7.33, given that $v_s(t) = 40 \sin(200t - 20^\circ)$ V.

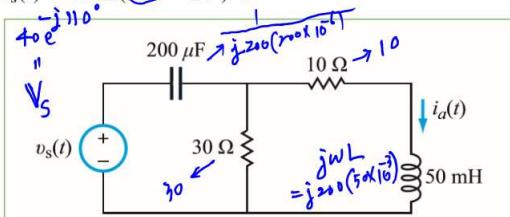
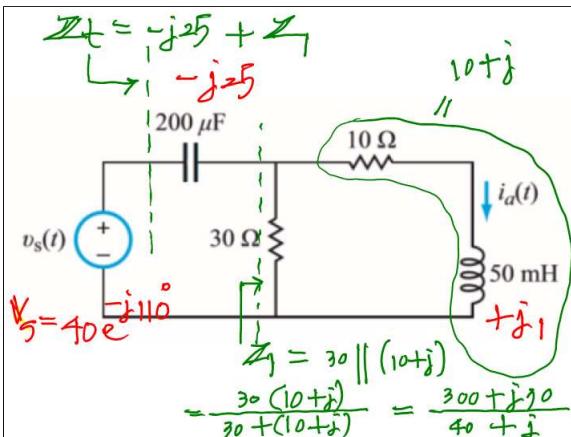
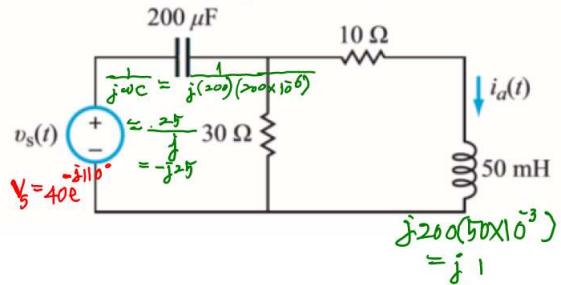


Figure P7.33: Circuit for Problem 7.33.

$$v_s = 40 \sin(200t - 20^\circ) = 40 \cos(200t - 20^\circ - 90^\circ)$$

$$= 40 \cos(200t - 110^\circ), \omega = 200$$



$$Z_t = -j25 + \frac{300 + j30}{40 + j}$$

$$= -j25(40+j) + 300 + j30$$

$$= \frac{-j1000 - j25 + 300 + j30}{40 + j}$$

$$= \frac{325 - j970}{40 + j}$$

$$I_t = \frac{40 e^{-j110}}{\left(\frac{325 - j970}{40 + j} \right)}$$

$$= \frac{(13.68 - j37.59)(40 + j)}{325 - j970}$$

$$\frac{I}{\alpha} = I_t \frac{30}{30 + (10 + j)}$$

$$= I_t \frac{30}{40 + j}$$

$$I_a = \frac{(-13.68 - j37.59)(40 + j)}{j25 - j970}$$

$$X \frac{30}{40 + j} = \frac{(-13.68)40 - j(37.59)(40) - j37.59(40) - j13.68}{j25(40) - j(j)970 + j325 - j970(40)}$$

$$= \frac{-547.2 + 37.59 - j(150.6 + 13.68)}{19000 + 970 + j(325 - 3880)} = \frac{-509.61 - j1517.28}{19370 - j3550}$$

$$I_a = \frac{-509.61 - j1517.28}{19370 - j3550}$$

$$= 0.1157 e^{-j(21.45^\circ + 90^\circ - 14.87^\circ)} = 0.1157 e^{-j14.58^\circ}$$

$$i_a(t) = 0.1157 \cos(200t + 14.58^\circ) = 0.1157 \cos(200t - 56.58^\circ)$$

*7.40 The circuit in Fig. P7.40 is in the phasor domain. Determine the following:

- (a) The equivalent input impedance Z at terminals (a, b).
- (b) The phasor current I , given that $V_s = 25\angle 45^\circ$ V.

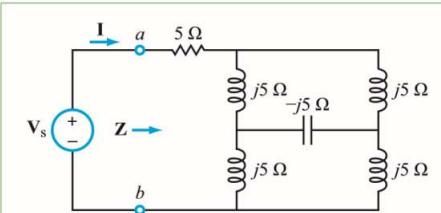
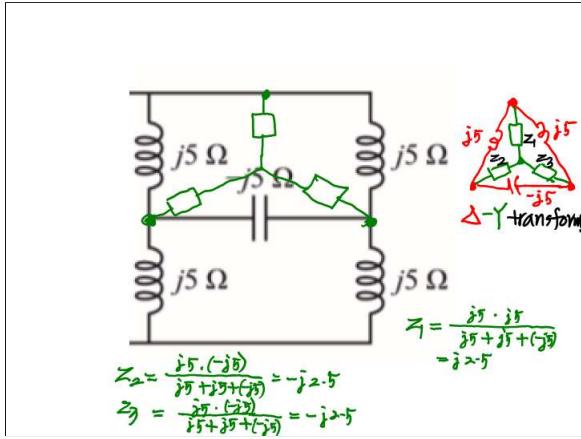


Figure P7.40: Circuit for Problem 7.40.



$$i(t) = 4 \cos(\omega t + 45^\circ)$$

$$I = \frac{V_s}{j1.25 + j1.25} = \frac{25 e^{j45^\circ}}{\sqrt{5^2 + 3.75^2} e^{j \tan^{-1} 0.75}} = \frac{25 j(45^\circ - 36^\circ)}{6.25 e^{j45^\circ}}$$

$$V_o = V_s \frac{1}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= V_s \frac{1}{j\omega RC + j^2 \omega^2 LC + 1}$$

$$\frac{|V_o|}{V_s} = \frac{1}{1 - (\frac{\omega}{\omega_0})^2 + j\omega RC}$$

$$\left| \frac{V_o}{V_s} \right| = \frac{1}{\sqrt{(1 - (\frac{\omega}{\omega_0})^2)^2 + (\omega RC)^2}}$$

At $\omega = \omega_0$, $\left| \frac{V_o}{V_s} \right| = \frac{1}{\omega_0 RC}$

At $\omega = \omega_3$,

$$\left| \frac{V_o}{V_s} \right|_{\omega=\omega_3} = \frac{1}{\sqrt{(1 - (\frac{\omega_3}{\omega_0})^2)^2 + (\omega_3 RC)^2}} = \left(\frac{1}{\sqrt{2}} \right) \omega_0 RC$$

squaring both sides \Rightarrow

$$(1 - (\frac{\omega_3}{\omega_0})^2)^2 + (\omega_3 RC)^2 = 2(\omega_0 RC)^2$$

Let $(\frac{\omega_3}{\omega_0})^2 = K$, then $(\omega_3 RC) = \sqrt{K}(\omega_0 RC)$

$$\frac{x_0 \log_{10} \left| \frac{V_o}{V_s} \right|}{\frac{1}{\sqrt{2}}} \leftarrow \log_{10} \left| \frac{V_o}{V_s} \right|^2$$

$$= 20 \left(\log_{10} 1 - \log_{10} \sqrt{2} \right) \quad \log_{10} 2^{\frac{1}{2}}$$

$$= 20 \left(0 - \frac{1}{2} \log_{10} 2 \right) \quad = \frac{1}{2} \log_{10} 2$$

$$= -3 \text{ dB}$$

$$(1 - K)^2 + K(\omega_0 RC)^2 = 2(\omega_0 RC)^2$$

$$1 - 2K + K^2 + K(\omega_0 RC)^2 - 2(\omega_0 RC)^2 = 0$$

$$K^2 - 2(\omega_0 RC)K + 1 - 2(\omega_0 RC)^2 = 0$$

$$K = \frac{2(\omega_0 RC) \pm \sqrt{[-2(\omega_0 RC)]^2 - 4(1 - 2(\omega_0 RC)^2)}}{2}$$

$$= \sqrt{4(\omega_0 RC)^2 + (\omega_0 RC)^4 - 4 + 8(\omega_0 RC)^2}$$

$$= \sqrt{4(\omega_0 RC)^2 + (\omega_0 RC)^4}$$

$$= \omega_0 RC \sqrt{4 + (\omega_0 RC)^2}$$

$$K = 1 - \frac{1}{2}(\omega_0 RC) \pm \omega_0 RC \sqrt{1 + \left(\frac{\omega_0 RC}{2}\right)^2}$$

For $\omega_0 RC = \frac{1}{2}$ case

(e.g.) $L = 1 \text{ mH}$, $C = 1 \text{ nF}$, $R = \frac{1}{2} \Omega$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^6, \quad \omega_0 RC = 10^6 \times \frac{1}{2} \times 10^{-6} = \frac{1}{2}$$

$$K = 1 - \frac{1}{2} \left(\frac{1}{2} \right) \pm \frac{1}{2} \sqrt{1 + \frac{1}{8}}$$

$$= 1 - \frac{1}{8} \pm \frac{1}{8} \sqrt{\frac{3}{2}} = \frac{7}{8} \pm \frac{3\sqrt{2}}{8} = \frac{7 \pm 4.4}{8}$$

$$= \frac{1}{8}(11.24, 2.76) = 1.43, 0.345$$

But $K = \left(\frac{\omega_3}{\omega_0} \right)^2 > 0$, thus $K = \left(\frac{\omega_3}{\omega_0} \right)^2$

$$\frac{1}{\omega_0 RC} = 1 \quad \omega_3+ = \sqrt{1.43} \omega_0 = 1.185 \omega_0$$

$$\omega_3- = \sqrt{0.345} \omega_0 = 0.587 \omega_0$$

$$0.587 \omega_0, \quad \omega_0 = 10^6, \quad \omega_{3dB} = 1.185 \omega_0, \quad \underline{\text{BW} = 0.6 \omega_0}$$

A Band Pass Filter

$$V_o(t) = V_m \cos(\omega t + \phi)$$

$$\frac{V_o}{V_s} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$$

$$= \frac{\omega RC}{\omega RC + j(\omega^2 LC - 1)}$$

$$\frac{V_o}{V_s} = \frac{\omega RC}{\omega RC + j(\omega^2 LC - 1)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\text{At } \omega = \omega_0, \quad \frac{V_o}{V_s} = \frac{\omega_0 RC}{\omega_0 RC + j\omega_0} = 1$$

At $(\omega_0 \pm \Delta\omega)$ freq., $\left| \frac{V_o}{V_s} \right| = \frac{1}{\sqrt{2}}$

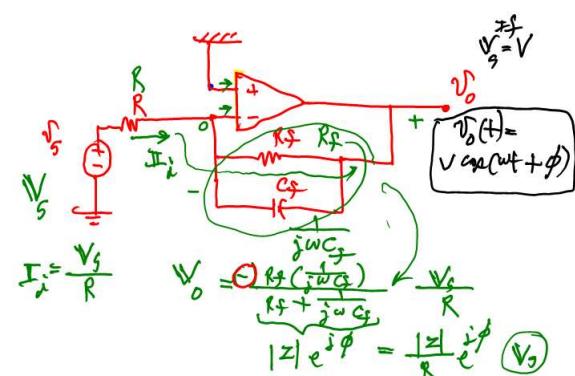
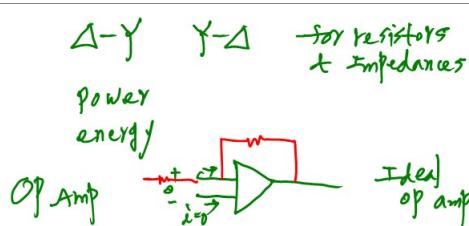
$$\left| \frac{\omega_3 RC}{\omega_3 + j[\frac{(\omega_3)^2}{\omega_0^2} - 1]} \right|$$

$$\begin{aligned}
 &= \frac{\omega_0 R C}{\sqrt{(\omega_0 R C)^2 + \left[\left(\frac{\omega_0}{\omega_0}\right)^2 - 1\right]^2}} = \frac{1}{\sqrt{2}} \\
 &\text{squaring both sides} \\
 &\omega_0^2 (\omega_0 R C)^2 = (\omega_0 R C)^2 + \left[\left(\frac{\omega_0}{\omega_0}\right)^2 - 1\right]^2 \\
 &\Rightarrow (\omega_0 R C)^2 = \left(\frac{\omega_0}{\omega_0}\right)^2 - \omega_0^2 (\omega_0 R C)^2 + 1 \\
 &\text{let } \left(\frac{\omega_0}{\omega_0}\right)^2 = K, \text{ then} \\
 &\omega_0^2 = K \omega_0^2 \\
 &(\omega_0 R C)^2 = K (\omega_0 R C)^2 \\
 &\Rightarrow K (\omega_0 R C)^2 = K^2 - 2K + 1
 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow K^2 - (2 + (\omega_0 R C)^2)K + 1 = 0 \\
 &K = \frac{(2 + (\omega_0 R C)^2) \pm \sqrt{(2 + (\omega_0 R C)^2)^2 - 4}}{2} \\
 &= 1 + \frac{(\omega_0 R C)^2}{2} \pm \sqrt{\frac{4 + 4(\omega_0 R C)^2 + (\omega_0 R C)^4}{4}} \\
 &= 1 + \frac{(\omega_0 R C)^2}{2} \pm \sqrt{(\omega_0 R C)^2 + \frac{1}{4}(\omega_0 R C)^4} \\
 &= \left(1 + \frac{(\omega_0 R C)^2}{2}\right) \pm \omega_0 R C \sqrt{1 + \left(\frac{\omega_0 R C}{2}\right)^2} \\
 &\left(\frac{\omega_0}{\omega_0}\right)^2 = K, \text{ thus } \underline{\omega_0 = \omega_0 \sqrt{K}}
 \end{aligned}$$

$$\begin{aligned}
 &\text{if } \omega_0 R C = \frac{1}{\sqrt{2}} R C = \lambda \\
 &K = 1 + \frac{(\omega_0 R C)^2}{2} \pm \omega_0 R C \sqrt{1 + \left(\frac{\omega_0 R C}{2}\right)^2} \\
 &= 1 + 2 \pm 2 \sqrt{1 + 1} \\
 &= 3 \pm 2\sqrt{2} = 5.828, 0.172 \\
 &\left(\frac{\omega_0}{\omega_0}\right)^2 = K, \quad \omega_0 = \omega_0 \sqrt{K} \\
 &= \omega_0 \sqrt{5.828}, \omega_0 \sqrt{0.172} \\
 &= 2.444\omega_0, 0.414\omega_0 \\
 &\text{e.g. } L = 1 \mu H, \quad C = 1 \mu F, \quad R = 2 \Omega \\
 &\omega_0 R C = \lambda, \quad \omega_0 = 10^6
 \end{aligned}$$

$$\begin{aligned}
 &\text{Ohm's Law} \quad \left\{ \begin{array}{l} V(t) = R I(t) \\ V(f) = R I(f) \\ V = R I \end{array} \right. \quad V(t) = R i(t) \\
 &\downarrow \Rightarrow \text{voltage division, current division} \\
 &R \quad \frac{1}{C f} \quad \text{Impedance } [-2] \\
 &I \quad j\omega h \quad \frac{1}{j\omega C} = -j\frac{1}{\omega C} \quad \text{Impedance } [2] \\
 &\text{Norton's equivalent ckt} \\
 &\text{Thévenin's equiv. ckt}
 \end{aligned}$$



$$\frac{R_f \frac{1}{j\omega C_f}}{R_f + \frac{1}{j\omega C_f}} \times e^{j\omega t} = \frac{R_f e^{j0^\circ}}{(1 + j R_f C_f \omega)}$$

Diagram showing a phasor in the complex plane with magnitude $|z|$ and phase angle $\phi = \tan^{-1}(\omega R_f C_f)$.

$$= \frac{R_f e^{j0^\circ}}{|z| e^{j\phi}} = \frac{R_f}{|z|} e^{j(0^\circ - \phi)}$$

$j^4 = -j^3 = 36.9^\circ$

$$= \sqrt{3^2 + 4^2} / \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$$

$$\tan^{-1}\frac{4}{3} = 53.1^\circ$$

$$\tan^{-1}\frac{4}{3} = 53.1^\circ$$

$$\tan^{-1} 1 = 45^\circ$$