ECE101 F19 Lecture 17, Nov. 21, 2019
HW #9 for Quiz 9 on Dec. 3
1) Prob. 6.32
2) 6.38
3) 6.40
4) 6.52
5) 6.54
6) 6.58
7) Prob 7.1
8) 7.2
9) 7.5
10) Quiz 9 Aug 6.09 09.15

26 T
AC Analysis
ODE
28 th
AC Analysis
Transformer

Quiz 9
5 th Overview

Figure 7-46. Complex power supply circuit.

\[ V(s) = \frac{A_1s + A_2}{s + \frac{1}{RC}} + \frac{B_1s + B_2}{s + \frac{1}{RC}} \]

\[ V(s) = \frac{A_1s + A_2}{s + \frac{1}{RC}} + \frac{B_1s + B_2}{s + \frac{1}{RC}} \]

\[ V(s) = \frac{A_1s + A_2}{s + \frac{1}{RC}} + \frac{B_1s + B_2}{s + \frac{1}{RC}} \]

\[ V(s) = \frac{A_1s + A_2}{s + \frac{1}{RC}} + \frac{B_1s + B_2}{s + \frac{1}{RC}} \]

\[ V(s) = \frac{A_1s + A_2}{s + \frac{1}{RC}} + \frac{B_1s + B_2}{s + \frac{1}{RC}} \]
\[ A_1 \cos \omega t + A_2 \sin \omega t \]
\[ = K \cos(\omega t - \theta) \]
\[ = \sqrt{A_1^2 + A_2^2} \cos(\omega t - \theta) \]
\[ \theta = \tan^{-1} \left( \frac{A_2}{A_1} \right) \]

\[ K \cos \theta = A_1 \]
\[ K \sin \theta = A_2 \]
\[ K = \sqrt{A_1^2 + A_2^2} \]

**Laplace**
\[ \mathcal{L} \left[ v_c(t) \right] = \frac{V_c}{s} \]
\[ s = \sigma + j\omega \]
\[ s \to j\omega \]

**Phasor Method**
\[ v_c = \frac{Z_c}{R + j\omega C} \]
\[ v_c = \frac{V_c}{1 + j\omega C} \]
\[ v_c = \frac{1}{1 + j\omega C} \]
\[ v_c = \frac{1}{1 + j\omega \tau} \cos(\omega t) + j\omega \tau \sin(\omega t) \]

\[ \exp(j \omega t - \theta) = \cos \omega t \cos \theta + \sin \omega t \sin \theta \]
\[ \frac{\tau}{\omega} = \sin \theta \]
\[ \sin \theta = 1 \cos \theta = 0 \]
\[ \theta = \frac{\pi}{2} \]
\[
\frac{Z_1}{Z_2} = \frac{|Z_1| e^{j\theta_1}}{|Z_2| e^{j\theta_2}} = \frac{|Z_1|}{|Z_2|} e^{j(\theta_1 - \theta_2)}
\]
\[
Z_1 Z_2 = |Z_1|^2 e^{j\theta_1} \cdot |Z_2|^2 e^{j\theta_2} = |Z_1| |Z_2| e^{j(\theta_1 + \theta_2)}
\]

\[
\theta = -\tan^{-1} \left( \frac{\text{Im}(a+jb)}{\text{Re}(a+jb)} \right) = -\tan^{-1}(\text{Im}(a+jb))
\]

This is exactly the same as the solution obtained by the Laplace method in steady state.

\[
\text{cos}(\omega t + \phi)
\]
\[
\frac{\text{cos}(\omega t - \phi)}{\text{cos}(\omega t - (-\phi))} = \frac{\text{cos}(\omega t - \phi)}{\text{cos}(\omega t + \phi)}
\]

\[
\text{Im}(z) = 0, \quad \text{Re}(z) = 0
\]
\[
x = |a| \cos \theta, \quad y = |a| \sin \theta
\]
\[
|a| = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x)
\]
Any cosinusoidally time-varying function \( x(t) \), representing a voltage or a current, can be expressed in the form

\[
x(t) = \Re\{X e^{j\omega t}\},
\]

(7.28)

where \( X \) is a time-independent function called the phasor counterpart of \( x(t) \). Thus, \( x(t) \) is defined in the time domain, while its counterpart \( X \) is defined in the phasor domain.

To distinguish phasor quantities from their time-domain counterparts, phasors are always represented by bold letters in this book.

\[
i(t) = \Re\{I e^{j\omega t}\} = \Re\{I e^{j(\omega t + \theta)}\}
\]

(7.35)

where \( I \) may be complex but, by definition, not a function of time. The derivative \( di/dt \) is given by

\[
\frac{di}{dt} = \Re\{ \frac{d}{dt} (I e^{j\omega t}) \} = \Re\{ j\omega I e^{j\omega t} \}
\]

(7.36)

\[
\frac{d}{dt} [\Im\{ I e^{j\omega t} \}] = \Im\{ j\omega I e^{j\omega t} \}
\]

\[
\frac{d}{dt} [\Im\{ I e^{j(\omega t + \theta)} \}] = \Im\{ j\omega I e^{j(\omega t + \theta)} \}
\]

Similarly,

\[
\int i \, dt = \Re\{ i e^{j\omega t} \} = \Re\{ i e^{j(\omega t + \theta)} \}
\]

(7.38)

or

\[
\int i \, dt = \frac{1}{j\omega} e^{j\omega t}
\]

(7.39)

\[
\int i \, dt = \frac{1}{j\omega} e^{j\omega t}
\]

(7.39)

\[
\int i \, dt = \frac{1}{j\omega} e^{j\omega t}
\]

The impedance \( Z \) of a circuit element is defined as the ratio of the phasor voltage across it to the phasor current entering through its plus (+) terminal.

\[
Z = \frac{V}{I} \quad (\Omega),
\]

(7.45)

and the unit of \( Z \) is the ohm (\( \Omega \)). For a resistor, Eq. (7.44) gives

\[
\frac{V_R}{I_R} = R.
\]

(7.46)
Phasors $V_L$ and $I_L$ are related to their time-domain counterparts by

$$v_L = \text{Re}(V_L e^{j \omega t}) \quad (7.48a)$$

and

$$i_L = \text{Re}(I_L e^{j \omega t}) \quad (7.48b)$$

Consequently,

$$\text{Re}(V_L e^{j \omega t}) = L \frac{d}{dt} \text{Re}(I_L e^{j \omega t}) = \text{Re}(j \omega L I_L e^{j \omega t}) \quad (7.49)$$

which leads to

$$V_L = j \omega L I_L \quad (7.50)$$

Hence, the impedance of an inductor $L$ is

$$Z_L = \frac{V_L}{I_L} = j \omega L \quad (7.51)$$

\[ \begin{array}{c}
\text{Capacitors} \\
\text{Since for a capacitor} \\
\frac{dQ}{dt} = C \frac{dv_c}{dt} \quad (7.52) \\
\text{It follows that in the phasor domain,} \\
\frac{dQ}{dt} = j \omega C \quad (7.53)
\end{array} \]

and the impedance of a capacitor $C$ is

$$Z_C = \frac{V_c}{I_c} = \frac{1}{j \omega C} \quad (7.54)$$

Because $Z_L$ and $Z_C$ are, respectively, directly and inversely proportional to $w$, $Z_L$ and $Z_C$ assume inverse roles as $w$ approaches zero and infinity.

\[ \text{In the phasor domain, a capacitor behaves like an open circuit at dc and like a short circuit at very high frequencies.} \]

\[ \text{Table 7-4: Summary of AC properties for } R, L, \text{ and } C \]

<table>
<thead>
<tr>
<th>Property</th>
<th>$R$</th>
<th>$L$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v = j \omega L i$</td>
<td>$v = L \frac{di}{dt}$</td>
<td>$i = C \frac{dv}{dt}$</td>
<td></td>
</tr>
<tr>
<td>$v = R i$</td>
<td>$V = R I$</td>
<td>$V = j \omega L I$</td>
<td></td>
</tr>
<tr>
<td>$Z = \frac{V}{I}$</td>
<td>$R$</td>
<td>$j \omega L$</td>
<td>$\frac{1}{j \omega C}$</td>
</tr>
<tr>
<td>dc equivalent</td>
<td>$R$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>High-frequency equivalent</td>
<td>$R$</td>
<td>Short circuit</td>
<td>Open circuit</td>
</tr>
<tr>
<td>Frequency response</td>
<td>$Z_L$</td>
<td>$Z_C$</td>
<td>$Z_L$</td>
</tr>
</tbody>
</table>

\[ X = \sqrt{R^2 + \left(\frac{1}{j \omega C}\right)^2} \]

\[ Z = \sqrt{R^2 + \left(\frac{1}{j \omega C}\right)^2} \]

\[ \theta = \tan\left(\frac{\omega}{\sqrt{R^2 + \left(\frac{1}{j \omega C}\right)^2}}\right) \]