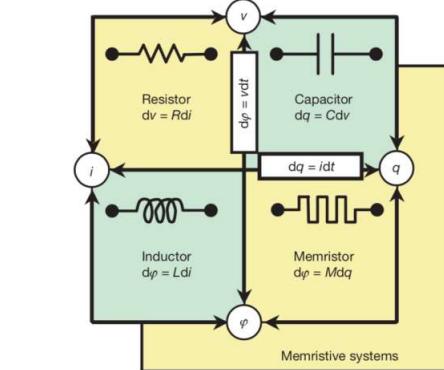
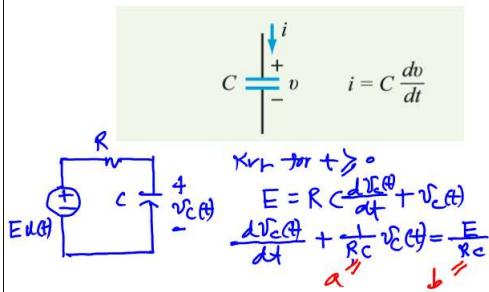


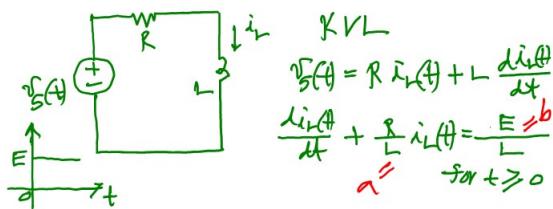
ECE101F19 Lecture 13 Nov 12, 2019

RC, RL + OP Amps



$$L \begin{cases} + \\ - \end{cases} \quad v_L = L \frac{di}{dt}$$

$$v_L(t) = L(sI_L(s) - I_L(0))$$



Energy stored in C, L

$$C \begin{cases} + \\ - \end{cases} \quad p(t) - v_C(t) i_C(t) = v_C(t) \left(C \frac{dv_C(t)}{dt} \right)$$

$$E(t) = \int_a^t p(t) dt = \int_0^t v_C(t) C \frac{dv_C(t)}{dt} dt$$

$$= C \int_0^t v_C^2(t) dv_C = \frac{1}{2} C \left[v_C^2(t) - v_C^2(0) \right]$$

$$= \frac{1}{2} C v_C^2(t) \quad \text{for } v_C(0) = 0$$

$$L \begin{cases} + \\ - \end{cases} \quad p(t) = v_L(t) i_L(t)$$

$$= L \frac{di_L(t)}{dt} i_L(t)$$

$$E(t) = \int_a^t p(t) dt = \int_0^t i_L(t) di_L(t)$$

$$= \frac{1}{2} L \left[i_L^2(t) - i_L^2(0) \right]$$

$$= \frac{1}{2} L i_L^2(t) \quad \text{for } i_L(0) = 0$$

In general

$$\frac{dx}{dt} + ax = b, \quad x(0-) \text{ given or calculated}$$

$$\text{For DC, } \lim_{t \rightarrow \infty} \frac{dx(t)}{dt} = 0, \quad ax(0) = b, \quad x(\infty) = \frac{b}{a}$$

$$x(t) = x(\infty) + (x(0) - x(\infty)) e^{-at}$$

Revisit L.e.

$$\frac{dx}{dt} + ax = b$$

Laplace transformation method:

$$\mathcal{L}\left[\frac{dx}{dt} + ax = b\right] \Rightarrow [sX(s) - x(0)] + aX(s) = \frac{b}{s}$$

$$\begin{cases} \mathcal{L}\left[\frac{dx(t)}{dt}\right] = sX(s) - x(0) \\ \mathcal{L}[ax(t)] = aX(s) \\ \mathcal{L}[b] = \frac{b}{s} \end{cases}$$

$$\Rightarrow (s+a)X(s) = \frac{b}{s} + x(0)$$

$$X(s) = \frac{b}{s(s+a)} + \frac{x(0)}{s+a}$$

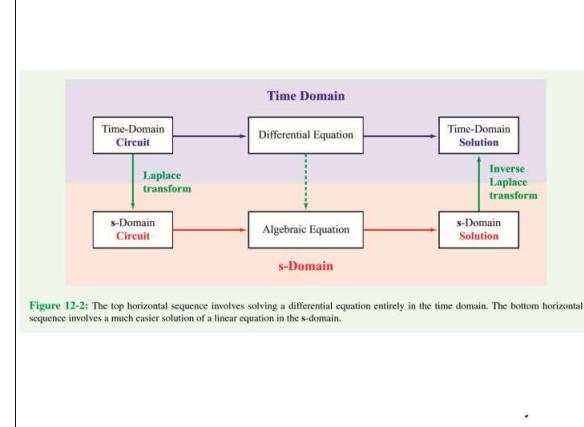


Figure 12-2: The top horizontal sequence involves solving a differential equation entirely in the time domain. The bottom horizontal sequence involves a much easier solution of a linear equation in the s-domain.

Solution Procedure: Laplace Transform

Step 1: The circuit is transformed to the Laplace domain—also known as the s-domain.

Step 2: In the s-domain, application of KVL and KCL yields a set of algebraic equations.

Step 3: The equations are solved for the variable of interest.

Step 4: The s-domain solution is transformed back to the time domain.

Table 12-1: Properties of the Laplace transform ($f(t) = 0$ for $t < 0^-$).

Property	$f(t)$	$F(s) = \mathcal{L}[f(t)]$
1. Multiplication by constant	$K f(t)$	$\leftrightarrow K F(s)$
2. Linearity	$K_1 f_1(t) + K_2 f_2(t)$	$\leftrightarrow K_1 F_1(s) + K_2 F_2(s)$
3. Time scaling	$f(at), \quad a > 0$	$\leftrightarrow \frac{1}{a} F\left(\frac{s}{a}\right)$
4. Time shift	$f(t-T) u(t-T)$	$\leftrightarrow e^{-Ts} F(s), \quad T \geq 0$
5. Frequency shift	$e^{-at} f(t)$	$\leftrightarrow F(s+a)$
6. Time 1st derivative	$f' = \frac{df}{dt}$	$\leftrightarrow sF(s) - f(0^-)$
7. Time 2nd derivative	$f'' = \frac{d^2f}{dt^2}$	$\leftrightarrow s^2 F(s) - sf(0^-) - f'(0^-)$
8. Time integral	$\int_0^t f(\tau) d\tau$	$\leftrightarrow \frac{1}{s} F(s)$
9. Frequency derivative	$t f(t)$	$\leftrightarrow -\frac{d}{ds} F(s)$
10. Frequency integral	$\frac{f(t)}{t}$	$\leftrightarrow \int_s^\infty F(s') ds'$

$$\begin{aligned} X(s) &= \frac{b}{s(s+a)} + \frac{x(0)}{s+a} \\ &= b \left[\frac{\frac{1}{s}}{s+a} + \frac{-\frac{1}{a}}{s+a} \right] + \frac{x(0)}{s+a} \\ &= \frac{\left(\frac{b}{s}\right)}{s+a} + \frac{x(0) - \frac{b}{a}}{s+a} \\ &\downarrow \mathcal{L}^{-1} \text{ inverse Laplace transform} \\ X(t) &= \frac{b}{a} + \left(x(0) - \frac{b}{a}\right) e^{-at} \\ &= x(\infty) + (x(0) - x(\infty)) e^{-at} \end{aligned}$$

Exercise 12-9: Convert the circuit in Fig. E12.9 into the s-domain.

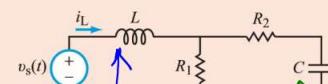
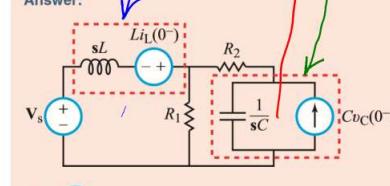


Figure E12.9

Answer:



(See CAD)

$$i_C = C \frac{dv_C}{dt}$$

Taking \int on both sides

$$I_C(s) = C [s V_C(s) - v_C(\omega)]$$

$$= s C V_C(s) - C v_C(\omega)$$

Inductor case (time invariant L)

$$V_L(s) = L I_L(s)$$

$$\frac{dV_L(s)}{dt} = \frac{d}{dt}(L I_L(s))$$

$$= L \frac{di_L}{dt}$$

$$V_L(s) = L(s I_L(s) - i_L(\omega))$$

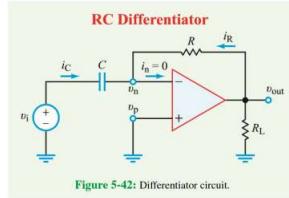


Figure 5-42: Differentiator circuit.

5-6.2 Ideal Op Amp Differentiator

The integrator circuit of Fig. 5-40 can be converted into the differentiator circuit of Fig. 5-42 by simply interchanging the locations of R and C . For the differentiator circuit, application of the voltage and current constraints leads to

$$i_C = C \frac{dv_i}{dt}, \quad i_R = \frac{v_{out}}{R}, \quad \text{and} \quad i_C = -i_R.$$

Consequently,

$$v_{out} = -RC \frac{dv_i}{dt}. \quad (5.131)$$

*5.48 Determine $i(t)$ for $t \geq 0$ given that the circuit in Fig. P5.48 had been in steady state for a long time prior to $t = 0$. Also, $I_0 = 5 \text{ A}$, $R_1 = 2 \Omega$, $R_2 = 10 \Omega$, $R_3 = 3 \Omega$, $R_4 = 7 \Omega$, and $L = 0.15 \text{ H}$.

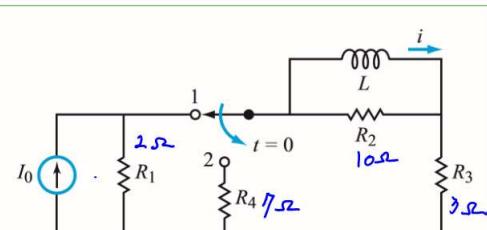
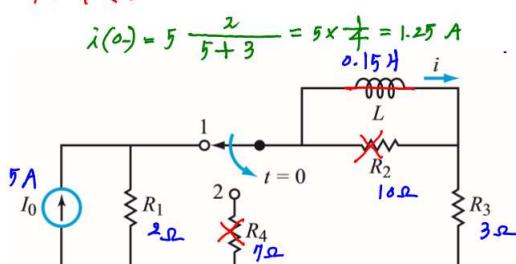


Figure P5.48: Circuit for Problem 5.48.

For $t < 0$



For $t \geq 0$

$$i(0) = 1.25 \text{ A}$$

$$i(0) = 1.25 \text{ A}$$

$$0.15 \frac{di}{dt} = -5 \lambda$$

$$\frac{\lambda}{dt} + \frac{5}{0.15} \lambda = 0$$

$$\lambda = \frac{10}{3} \Rightarrow i(0) = 0$$

$$\begin{aligned} \hat{I}(t) &= \hat{I}(\infty) + (\hat{I}(0) - \hat{I}(\infty)) e^{-\frac{10}{2}t} \\ &= 1.25 e^{-\frac{10}{2}t} \end{aligned}$$

*5.63 Relate $i_{out}(t)$ to $v_i(t)$ in the circuit of Fig. P5.63. Evaluate it for $v_C(0) = 3$ V, $R = 10 \text{ k}\Omega$, $C = 50 \mu\text{F}$, and $v_i(t) = 9 u(t)$ V.

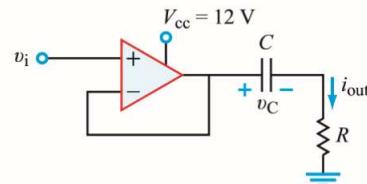
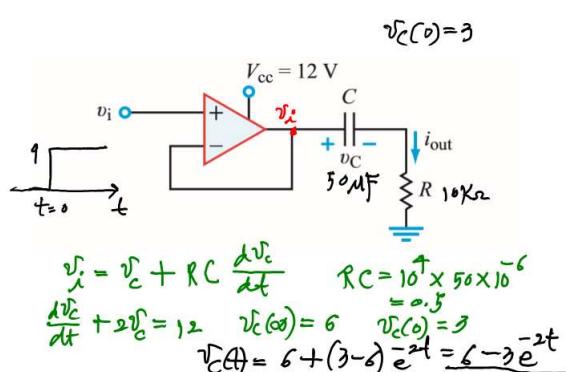


Figure P5.63: Circuit for Problem 5.63.



5.68 The two-stage op-amp circuit in Fig. P5.68 is driven by an input step voltage given by $v_i(t) = 10 u(t)$ mV. If $V_{cc} = 10$ V for both op amps and the two capacitors had no charge prior to $t = 0$, determine and plot:

$v_C(0) = 0 \text{ for both capacitors}$

(a) $v_{out_1}(t)$ for $t \geq 0$;

(b) $v_{out_2}(t)$ for $t \geq 0$.

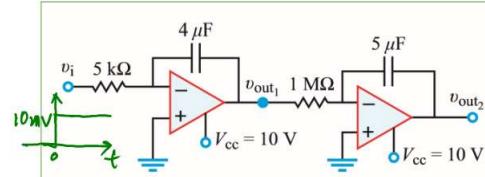
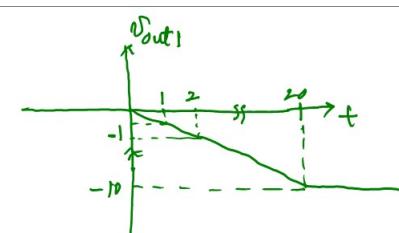
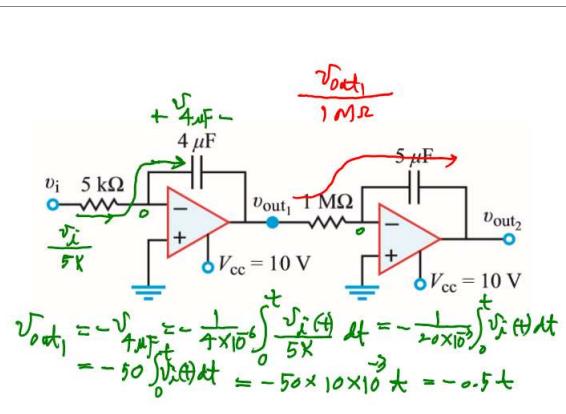


Figure P5.68: Op-amp circuit for Problem 5.68.



For $t < 20\pi$,

$$v_{out2} = -\frac{1}{5 \times 10^6} \int_{0}^{t-0.5t} \frac{dt}{10^6} dt$$

$$= +0.1 \int_0^t dt + 4 = +0.1 \times \frac{t^2}{2} = 0.05 t^2$$

$$v_{out2} = 10 \text{ at } t = \sqrt{\frac{10}{0.05}} = 10\sqrt{2} \text{ sec}$$

(saturation)

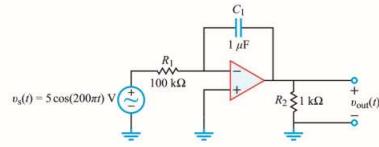
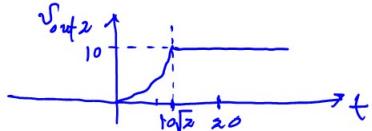


Figure m5.6 Circuit for Problem m5.6.

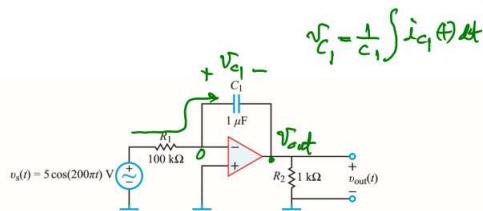
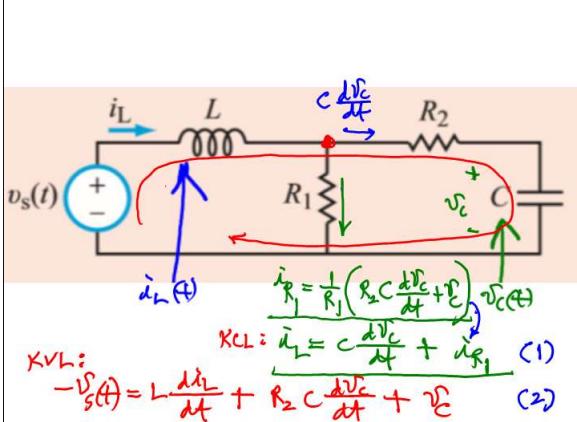
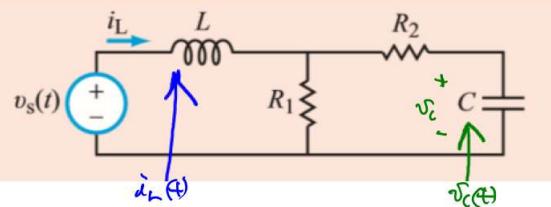


Figure m5.6 Circuit for Problem m5.6.

$$i_R1 = i_C = \frac{v_s - 0}{100k} = \frac{5 \cos(200\pi t)}{100k}$$

$$v_{out} = -v_C = -\frac{1}{10^6} \int_0^t \frac{v_s}{100k} dt = -10 \int_0^t \cos(200\pi t) dt$$

$$= \frac{-50}{200\pi} \sin 200\pi t$$



$i_{R1} = \frac{1}{R_1} (R_2 C \frac{dv_C}{dt} + v_C)$

KCL: $\frac{i_L}{R_1} = C \frac{dv_C}{dt} + i_{R1}$ (1)

$-v_s(t) = L \frac{di_L}{dt} + R_2 C \frac{dv_C}{dt} + v_C$ (2)

$\Downarrow -v_s(t) = L \frac{di_L}{dt} + R_2 C \frac{dv_C}{dt} + \frac{1}{R_1} v_C(t) + v_C$

$$y(t) = L_C \left(1 + \frac{R_2}{R_1}\right) \frac{d^2 C}{dt^2} + \left(\frac{C}{R_1} + R_2\right) \frac{dC}{dt} + V_C$$

$$a \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + c x = f(t)$$

2nd order differential eq.

$$\frac{d^2 x}{dt^2} + \frac{b}{a} \frac{dx}{dt} + \frac{c}{a} x = \frac{1}{a} f(t)$$

$$\boxed{\frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_2 x = g(t)}$$

$$L \downarrow$$

$$2x''(s) - s x'(s) - x(s) + a_1 [f(x(s)) - x(s)] + a_2 x(s) = g(s)$$

$$\begin{aligned}
 & \Rightarrow s^2 X(s) + a_1 s X(s) + a_2 X(s) \\
 &= \mathcal{L}(f(s)) + s x(a) + x'(a) + a_1 x(a) \\
 X(s) &= \frac{\mathcal{L}(f(s)) + (s+a_1)x(a) + x'(a)}{s^2 + a_1 s + a_2} \\
 & \quad \text{Find } x(t) \downarrow s^{-1}
 \end{aligned}$$

