

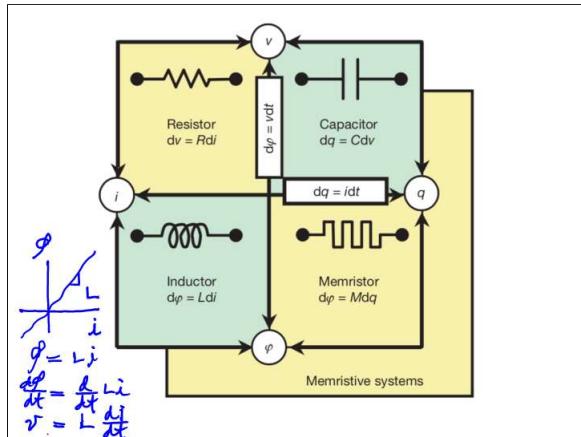
$$\frac{dV_c(t)}{dt} + \alpha V_c(t) = b$$

$$\text{As } t \rightarrow \infty, \quad 0 + \alpha V_c(\infty) = b$$

$$V_c(\infty) = -\frac{b}{\alpha} = -\frac{E}{RC} / \left( \frac{+}{R_C} \right) = E$$

$$V_c(t) = V_c(0) + (V_c(\infty) - V_c(0)) e^{-\alpha t}$$

$$= E + (V_c(0) - E) e^{-\frac{1}{RC} t}$$



$$V_L = L \frac{di}{dt}$$

$$V_L(\omega) = L \left( s I_L(s) - i_L(\omega) \right)$$

$$V_L(t) = R i_L(t) + L \frac{di_L(t)}{dt}$$

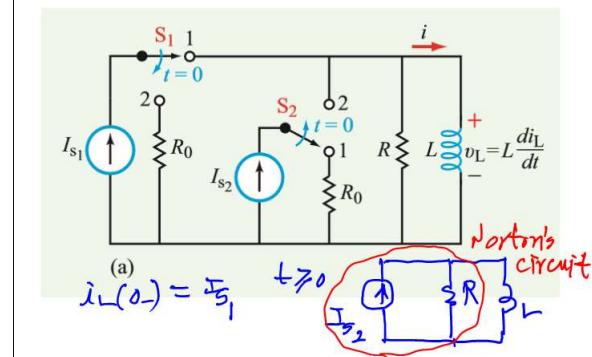
$$\frac{di_L(t)}{dt} + \frac{R}{L} i_L(t) = \frac{E}{L} \quad \text{for } t \geq 0$$

$$i_L(t) = [i_L(\infty) + [i_L(0) - i_L(\infty)] e^{-t/\tau}] u(t), \quad (5.107)$$

(switch action at  $t = 0$ )

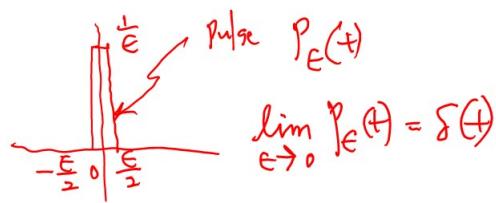
with time constant  $\tau = L/R$ . For the specific circuit

$$e^{-t/\tau} = e^{-\frac{R}{L} t}$$



$$C \frac{d}{dt} \left( \frac{1}{V_C} \right) = i_C$$

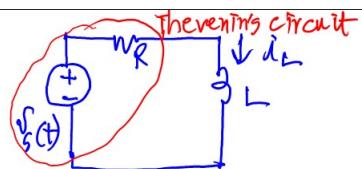
$$V_C = \frac{1}{C} \int_{t_0}^t i_C dt$$



$$L \frac{d}{dt} \left( \frac{1}{V_L} \right) = i_L$$

$$i_L = \frac{1}{L} \int_{t_0}^t V_L dt$$

$i$  remain same

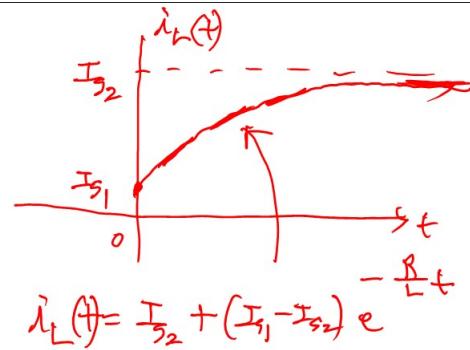


$$V_s(t) = R i_{S2} = E$$

for which

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty)) e^{-\frac{R}{L}t}$$

$$i_L(\infty) = \frac{E}{R} = i_{S2}, \quad i_L(0) = i_{S1}$$



#### Parallel RL Circuit Solution

1: If switch action is at  $t = 0$ , analyze circuit at  $t = 0^-$  (by replacing  $L$  with a short circuit) to determine initial conditions  $i_L(0^-)$  and  $v_R(0^-)$ . Use this information to determine  $i_L(0)$  and  $i_R(0)$  at  $t$  immediately after the switch action. Remember that the current through an inductor cannot change instantaneously (between  $t = 0^-$  and  $t = 0$ ), but the voltage can.

2: Analyze the circuit to determine  $i_L(\infty)$ , the current through the inductor long after the switch action.

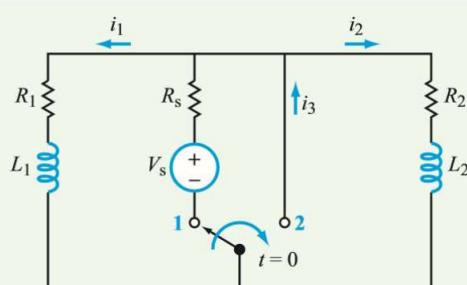
3: Determine the time constant  $\tau = L/R$ .

4: Incorporate the information obtained in the previous three steps in Eq. (5.107):

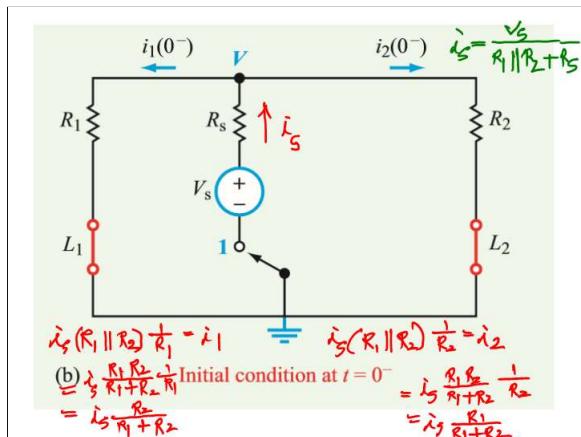
$$i_L(t) = [i_L(\infty) + [i_L(0) - i_L(\infty)] e^{-t/\tau}] u(t).$$

5: If the switch action is at  $t = T_0$  instead of  $t = 0$ , replace 0 with  $T_0$  everywhere and use Eq. (5.108):

$$i_L(t) = \left\{ i_L(\infty) + [i_L(T_0) - i_L(\infty)] e^{-\frac{(t-T_0)}{\tau}} \right\} u(t-T_0)$$



(a) Circuit with 2 inductors



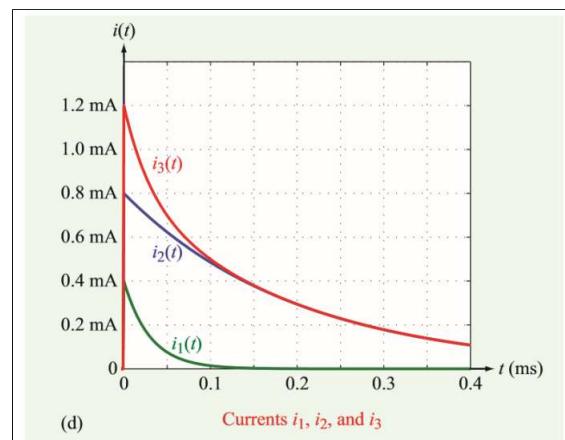
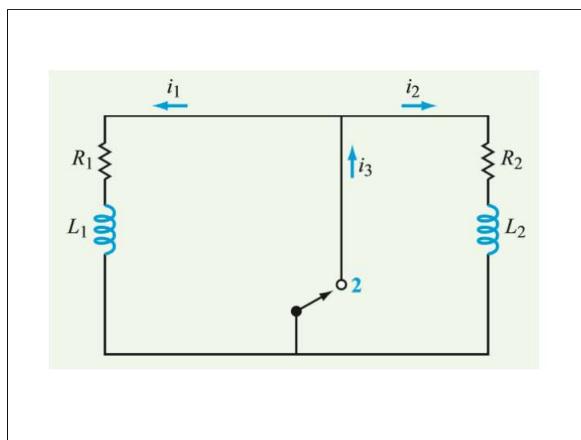
For  $t > 0$

$$R_1 i_1 + L_1 \frac{di_1}{dt} = 0 \quad R_2 i_2 + L_2 \frac{di_2}{dt} = 0$$

$$i_1(\infty) = 0 \quad i_2(\infty) = 0$$

$$i_1(t) = i_1(0) e^{-\frac{R_1 t}{L_1}} + i_2(t) e^{-\frac{R_2 t}{L_2}}$$

$$i_2(t) = i_1(t) + i_2(t)$$



$$\frac{di_L}{dt} + \alpha i_L = b$$

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty)) e^{-\alpha t}$$

$$= i_L(\infty) + (i_L(0) - i_L(\infty)) e^{-\frac{R}{L} t} +$$

$$\alpha = \frac{R}{L} \quad A, \rightarrow \infty \text{ (inductor shorted)}$$

$$b = \frac{E}{R}$$

$$\frac{di_L(t)}{dt} + \alpha i_L(t) = b$$

$$i_L(t) = \frac{b}{\alpha} = \frac{E}{R} = \frac{E}{\frac{R}{L}}$$

$$i_L(t) = \frac{E}{R} + (\lambda_L(0) - \frac{E}{R}) e^{-\frac{R}{L} t}$$

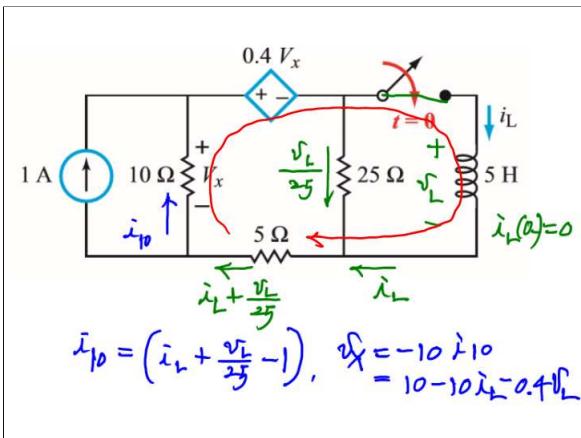
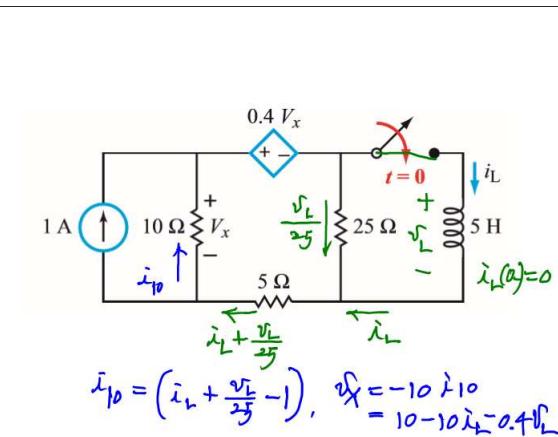
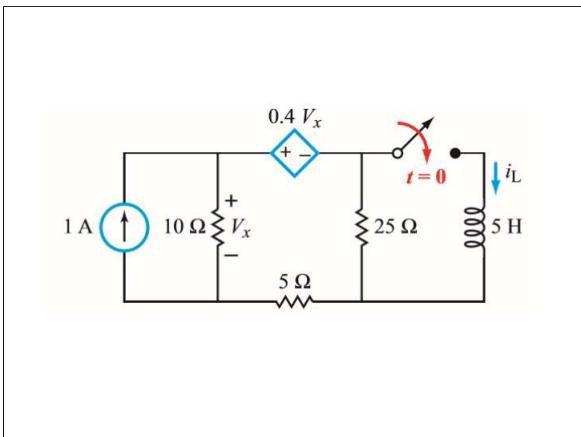
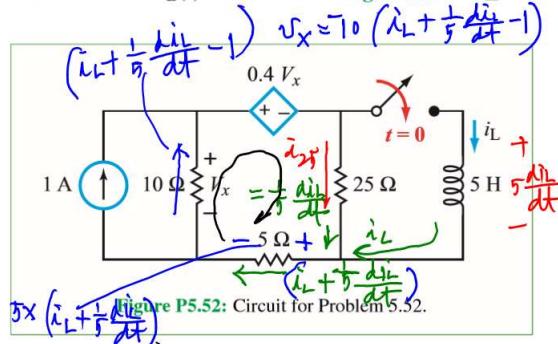
- HW 7 → Quiz 7 on Nov 14
- |               |                |
|---------------|----------------|
| 1) Prob 5. 25 | 9) Prob. 5. 60 |
| 2) 5. 31      | 10) 5. 65      |
| 3) 5. 33      |                |
| 4) 5. 36      |                |
| 5) 5. 45      |                |
| 6) 5. 48      |                |
| 7) 5. 50      |                |
| 8) 5. 56      |                |

$$V_L = V_{ab} \frac{di}{dt}$$

$$= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} = (L_1 + L_2 + L_3) \frac{di}{dt}$$

$$i = \frac{1}{L_1} \int i_{ab} dt + \frac{1}{L_2} \int i_{ab} dt + \frac{1}{L_3} \int i_{ab} dt = \left( \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right) i_{ab} t$$

5.52 Determine  $i_L(t)$  in the circuit of Fig. P5.52 for  $t \geq 0$ .



$$\text{KVL} : -V_x + 0.4 V_x + V_L + 5 \left( i_L + \frac{V_L}{25} \right) = 0 \quad (1)$$

$$V_x = 10 - 10 i_L - 0.4 V_L \quad (2)$$

$$(2) \rightarrow (1) \Rightarrow -0.6 (10 - 10 i_L - 0.4 V_L) + V_L + 5 i_L + 0.2 V_L = 0$$

$$-6 + 6 i_L + 0.2 V_L + V_L + 5 i_L + 0.2 V_L = 0$$

$$1.44 V_L + 11 i_L = 6 \quad (1)'$$

$$\text{where } V_L = 5 \frac{di}{dt} \quad (3)$$

$$(3) \rightarrow (1)' \Rightarrow 7.2 \frac{di}{dt} + 11 i_L = 6$$

$$\frac{di}{dt} + \frac{11}{7.2} i_L = \frac{6}{7.2}$$

$$\frac{d\dot{i}_L}{dt} + \frac{1}{\gamma_2} \dot{i}_L = -\frac{6}{\gamma_2}$$

$$\dot{i}_L(\omega) = -\frac{6}{\gamma_2}$$

$$\dot{i}_L(t) = -\frac{6}{\gamma_2} (1 - e^{-\frac{1}{\gamma_2} t})$$

### Combining In-Series Capacitors

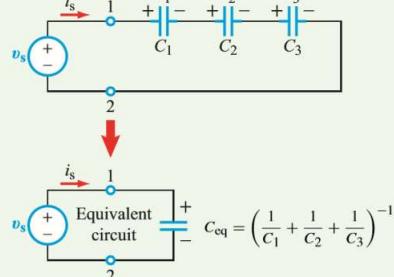
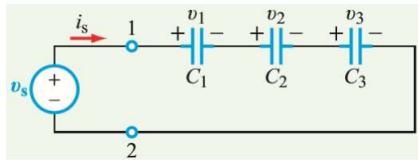


Figure 5-16: Capacitors in series.



### Combining In-Parallel Capacitors

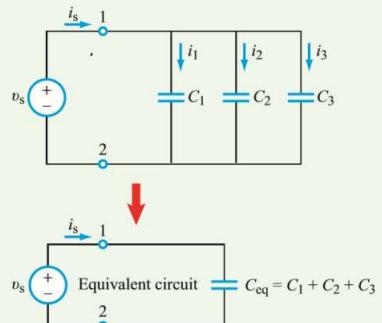


Figure 5-17: Capacitors in parallel.

### Voltage Division

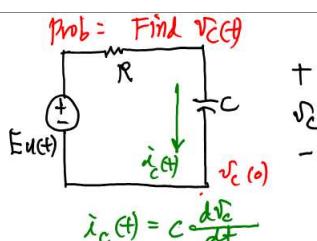
(a)  $v_1 = \left( \frac{R_1}{R_1 + R_2} \right) v_s$

$v_2 = \left( \frac{R_2}{R_1 + R_2} \right) v_s$

(b)  $v_1 = \left( \frac{C_2}{C_1 + C_2} \right) v_s$

$v_2 = \left( \frac{C_1}{C_1 + C_2} \right) v_s$

Figure 5-19: Voltage-division rules for (a) in-series resistors and (b) in-series capacitors.



Current will stop only when  $v_c = E$

$$KVL \quad E u(t) = R i_c(t) + v_c(t)$$

$$= R C \frac{dv_c(t)}{dt} + v_c(t)$$

$$For t \geq 0 \quad E = R C \frac{dv_c}{dt} + v_c$$

In general

$$RC \frac{dV_c}{dt} + V_c = E$$

$$\frac{dV_c}{dt} + \frac{1}{RC} V_c = \frac{E}{RC} \quad \alpha = \frac{1}{RC}$$

$$\frac{dV_c}{dt} + \alpha V_c = b$$

$$V_c(t) = V_c(\infty) + (V_c(0) - V_c(\infty)) e^{-\frac{t}{RC}}$$

Revisit L.e.

$$\frac{d^2V_c}{dt^2} + \alpha^2 V_c = b$$

Laplace transformation method:

$$\mathcal{L}\left[\frac{d^2V_c}{dt^2} + \alpha^2 V_c = b\right] \Rightarrow [5V_c(s) - V_c(0)] + \alpha V_c(s) = \frac{b}{s}$$

$$\begin{cases} \mathcal{L}\left[\frac{dV_c}{dt}\right] = sV_c(s) - V_c(0) \\ \mathcal{L}[aV_c(s)] = aV_c(s) \\ \mathcal{L}[b] = \frac{b}{s} \end{cases}$$

$$\Rightarrow (s+\alpha)V_c(s) = \frac{b}{s} + V_c(0)$$

$$V_c(s) = \frac{b}{s(s+\alpha)} + \frac{V_c(0)}{s+\alpha}$$

$$\begin{aligned} \frac{0s+b}{s(s+a)} &= \frac{A}{s} + \frac{B}{s+a} \\ &\approx \frac{A(s+a) + Bs}{s(s+a)} = \frac{(A+B)s + Aa}{s(s+a)} \\ A+B &= 0 \\ Aa &= b \rightarrow A = \frac{b}{a}, B = -\frac{b}{a} \\ \frac{b/a}{s} - \frac{b/a}{s+a} &= \frac{b/a}{a} - \frac{b/a}{a} e^{-at} \end{aligned}$$

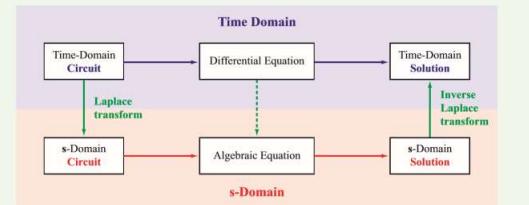


Figure 12-2: The top horizontal sequence involves solving a differential equation entirely in the time domain. The bottom horizontal sequence involves a much easier solution of a linear equation in the s-domain.

### Solution Procedure: Laplace Transform

**Step 1:** The circuit is transformed to the Laplace domain—also known as the s-domain.

**Step 2:** In the s-domain, application of KVL and KCL yields a set of algebraic equations.

**Step 3:** The equations are solved for the variable of interest.

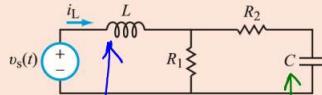
**Step 4:** The s-domain solution is transformed back to the time domain.

Table 12-1: Properties of the Laplace transform ( $f(t) = 0$  for  $t < 0^-$ ).

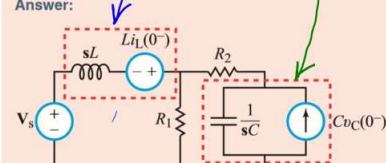
Property	$f(t)$	$F(s) = \mathcal{L}[f(t)]$
1. Multiplication by constant	$K f(t)$	$\rightarrow K F(s)$
2. Linearity	$K_1 f_1(t) + K_2 f_2(t)$	$\rightarrow K_1 F_1(s) + K_2 F_2(s)$
3. Time scaling	$f(at), \quad a > 0$	$\rightarrow \frac{1}{a} F\left(\frac{s}{a}\right)$
4. Time shift	$f(t-T) u(t-T)$	$\rightarrow e^{-Ts} F(s), \quad T \geq 0$
5. Frequency shift	$e^{-at} f(t)$	$\rightarrow F(s+a)$
6. Time 1st derivative	$f' = \frac{df}{dt}$	$\rightarrow sF(s) - f(0^-)$
7. Time 2nd derivative	$f'' = \frac{d^2f}{dt^2}$	$\rightarrow s^2 F(s) - sf(0^-) - f'(0^-)$
8. Time integral	$\int_0^t f(\tau) d\tau$	$\rightarrow \frac{1}{s} F(s)$
9. Frequency derivative	$t f(t)$	$\rightarrow -\frac{d}{ds} F(s)$
10. Frequency integral	$\frac{f(t)}{t}$	$\rightarrow \int_s^\infty F(s') ds'$

$$\begin{aligned}
 V_c(s) &= \frac{b}{s(s+a)} + \frac{V_c(0)}{s+a} \\
 &= b \left[ \frac{\frac{1}{s}}{s+a} + \frac{-\frac{1}{a}}{s+a} \right] + \frac{V_c(0)}{s+a} \\
 &= \frac{(\frac{b}{s})}{s} + \frac{V_c(0) - \frac{b}{a}}{s+a} \\
 &\downarrow \text{Inverse Laplace Transform} \\
 V_c(t) &= \frac{b}{a} + \left( V_c(0) - \frac{b}{a} \right) e^{-at} \\
 &= V_c(\infty) + (V_c(0) - V_c(\infty)) e^{-at}
 \end{aligned}$$

**Exercise 12-9:** Convert the circuit in Fig. E12.9 into the s-domain.



Answer:



(See CAD)

$$\begin{aligned}
 &C \frac{d^2v_c}{dt^2} \\
 &I_c = C \frac{d^2v_c}{dt^2} \\
 &\text{Taking } L \text{ on both sides} \\
 &I_c(s) = C [s V_c(s) - v_c(0)] \\
 &= C s V_c(s) - C v_c(0) \\
 &\frac{1}{Cs} \frac{d^2v_c}{dt^2} + V_c(s) = C v_c(0)
 \end{aligned}$$

$$\begin{aligned}
 &\text{Inductor case (time invariant } L) \\
 &I_L = \int i_L dt \\
 &V_L(s) = L I_L(s) = L \int i_L dt \\
 &= L \frac{di_L}{dt} \\
 &+ \frac{1}{L} \frac{dV_L}{dt} = V_L(s) = L(s I_L(s) - i_L(0))
 \end{aligned}$$

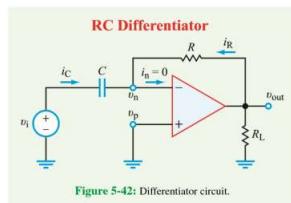


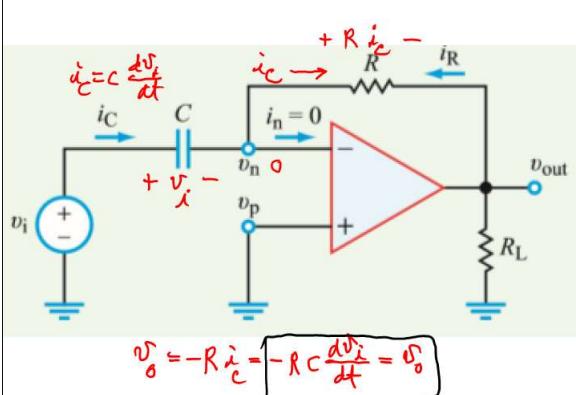
Figure 5-42: Differentiator circuit.

**5-6.2 Ideal Op Amp Differentiator**  
The integrator circuit of Fig. 5-40 can be converted into the differentiator circuit of Fig. 5-42 by simply interchanging the locations of  $R$  and  $C$ . For the differentiator circuit, application of the voltage and current constraints leads to

$$i_C = C \frac{dv_i}{dt}, \quad i_R = \frac{v_{out}}{R}, \quad \text{and} \quad i_C = -i_R.$$

Consequently,

$$v_{out} = -RC \frac{dv_i}{dt}. \quad (5.131)$$



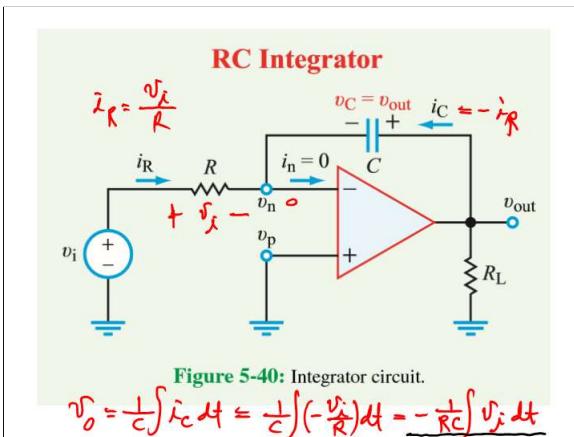
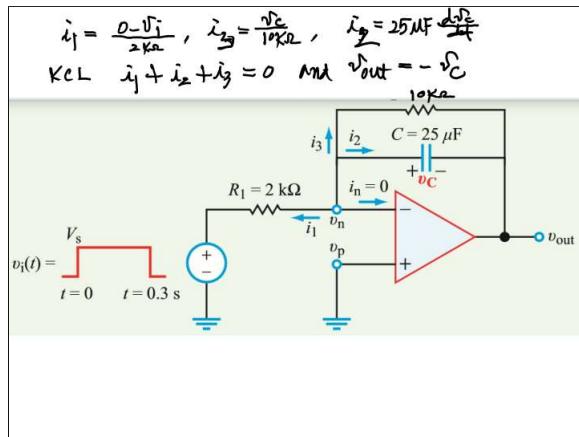
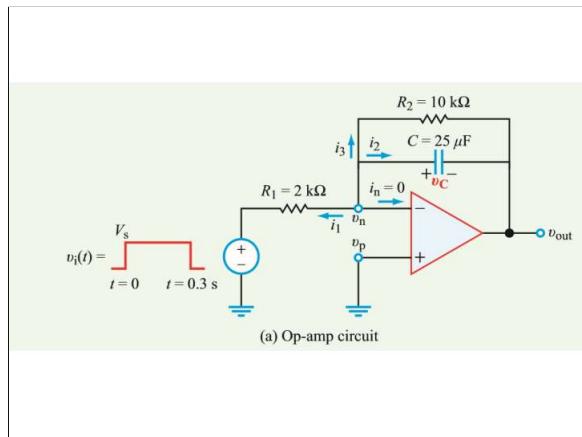


Figure 5-40: Integrator circuit.



$$\frac{-V_i}{2k\Omega} + \frac{V_c}{10k\Omega} + 25 \times 10^{-6} \frac{dv_c}{dt} = 0$$

divide by  $25 \times 10^{-6}$   $\Rightarrow$

$$\frac{-2V_i}{2 \times 10^3 \times 25 \times 10^{-6}} + \frac{V_c}{10^3 \times 25 \times 10^{-6}} + \frac{dv_c}{dt} = 0$$

$$\frac{dv_c}{dt} + 4V_c = 20V_i$$

For  $t < 0.3s$ ,

$$\frac{dv_c}{dt} + 4V_c = 20V_i$$

$$V_c(t) = V_c(\infty) + (V_c(0) - V_c(\infty)) e^{-4t}$$

$$V_c(\infty) = 5V_s \quad 5V_s(1 - e^{-4t})$$

$$V_c(0) = 0$$

