

ECE101F19 Lecture 10 Oct. 31, 2019

OP Amps for Signal Processing (pp 209-234)

H W # 6 for Quiz 6 on Nov. 7

[1] Prob. 5.3 (chap 5 prob on  
[2] Prob. 5.7 RC circuits)

[3] Prob. 5.9

[4] Prob. 5.15

RZ 4

[5] Prob. 5.17

Avg 5.73  
or 2.70

[6] Prob. 5.19

[7] Prob. 5.21

[8] Prob. 5.22

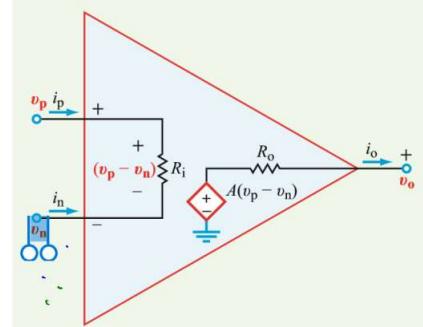
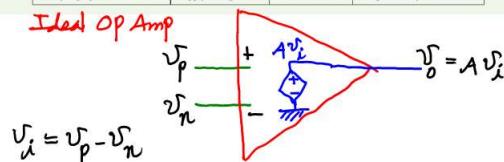


Figure 4-6: Equivalent circuit model for an op amp operating in the linear range ( $v_o \leq |V_{cc}|$ ). Voltages  $v_p$ ,  $v_n$ , and  $v_o$  are referenced to ground.

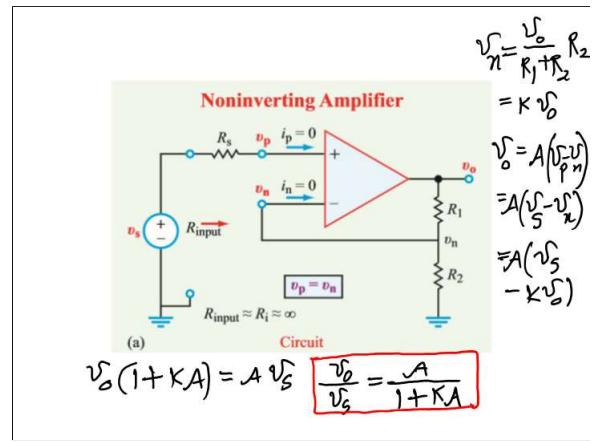
Table 4-1: Characteristics and typical ranges of op-amp parameters. The rightmost column represents the values assumed by the ideal op-amp model.

Op-Amp Characteristics	Parameter	Typical Range	Ideal Op Amp
Linear input-output response	Open-loop gain $A$	$10^4$ to $10^8$ (V/V)	$\infty$
High input resistance	Input resistance $R_i$	$10^6$ to $10^{13}$ $\Omega$	$\infty$ $\Omega$
Low output resistance	Output resistance $R_o$	1 to $100$ $\Omega$	0 $\Omega$
Very high gain	Supply voltage $V_{cc}$	5 to 24 V	As specified by manufacturer

Ideal Op Amp



$$v_o = v_p - v_n$$



$$\frac{v_o}{v_s} = \frac{A}{1 + KA}, \quad K = \frac{R_2}{R_1 + R_2}$$

when  $A \rightarrow \infty$  (very large)

$$\frac{v_o}{v_s} = \frac{1}{K + \frac{1}{A}} \Big|_{K \gg \infty} = \frac{1}{K} = \frac{R_1 + R_2}{R_2}$$

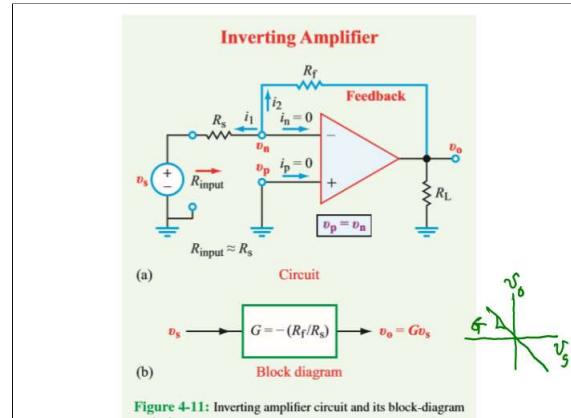
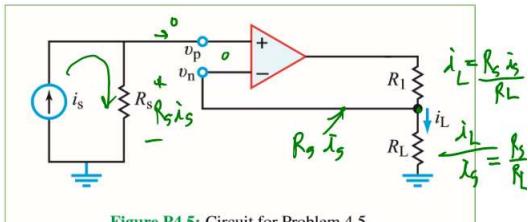


Figure 4-11: Inverting amplifier circuit and its block-diagram equivalent.

4.5 For the op-amp circuit shown in **Fig. P4.5**:

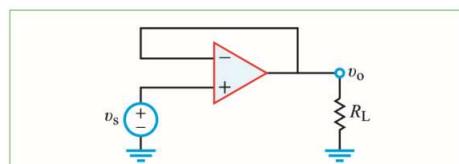
- Use the model given in **Fig. 4-6** to develop an expression for the current gain  $G_i = i_L/i_s$ .
- Simplify the expression by applying the ideal op-amp model (taking  $A \rightarrow \infty$ ,  $R_i \rightarrow \infty$ , and  $R_o \rightarrow 0$ ).



**Figure P4.5:** Circuit for Problem 4.5.

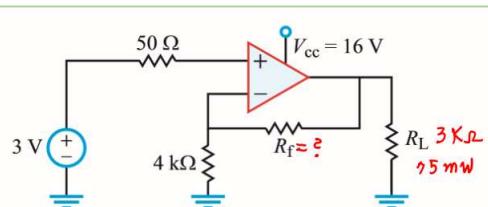
4.7 For the circuit in **Fig. P4.7**:

- Use the op-amp equivalent-circuit model to develop an expression for  $G = v_o/v_s$ .
- Simplify the expression by applying the ideal op-amp model parameters, namely  $A \rightarrow \infty$ ,  $R_i \rightarrow \infty$ , and  $R_o \rightarrow 0$ .

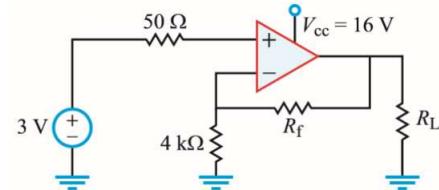


**Figure P4.7:** Circuit for Problem 4.7.

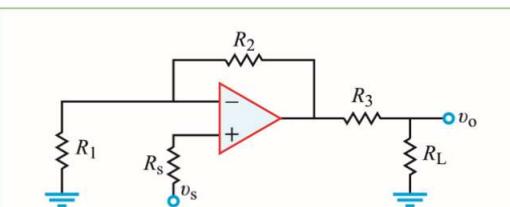
\*4.9 The supply voltage of the op amp in the circuit of **Fig. P4.9** is 16 V. If  $R_L = 3 \text{ k}\Omega$ , assign a resistance value to  $R_f$  so that the circuit would deliver 75 mW of power to  $R_L$ .



**Figure P4.9:** Circuit for Problem 4.9.

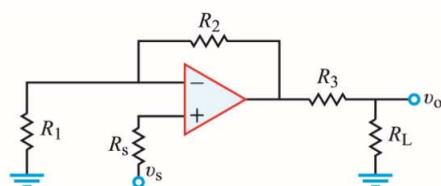


\*4.13 Obtain an expression for the voltage gain  $G = v_o/v_s$  for the circuit in **Fig. P4.13**.



**Figure P4.13:** Circuit for Problem 4.13.

$$v_o = f(v_s)$$



**4.17** Determine  $v_o$  across the  $10\text{ k}\Omega$  resistor in the circuit of Fig. P4.17.

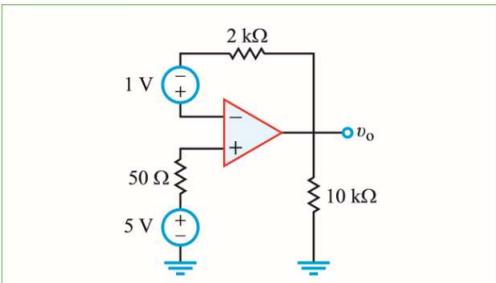


Figure P4.17: Circuit for Problem 4.17.

**4.22** The circuit in Fig. P4.22 uses a potentiometer whose total resistance is  $R = 10\text{ k}\Omega$  with the upper section being  $\beta R$  and the bottom section  $(1 - \beta)R$ . The stylus can change  $\beta$  from 0 to 0.9. Obtain an expression for  $G = v_o/v_s$  in terms of  $\beta$  and evaluate the range of  $G$  (as  $\beta$  is varied over its own allowable range).

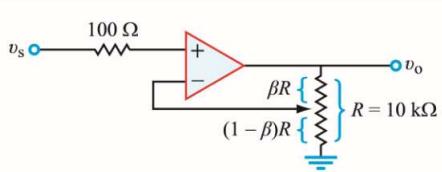


Figure P4.22: Circuit for Problem 4.22.

**4.23** For the circuit in Fig. P4.23, obtain an expression for voltage gain  $G = v_o/v_s$ .

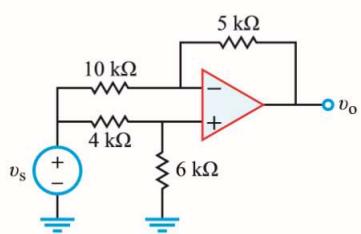


Figure P4.23: Circuit for Problem 4.23.

**4.29** Relate  $v_o$  in the circuit of Fig. P4.29 to  $v_s$  and specify the linear range of  $v_s$ . Assume  $V_0 = 0$ .

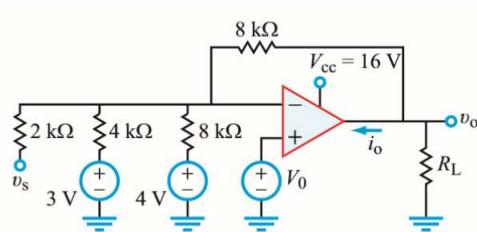
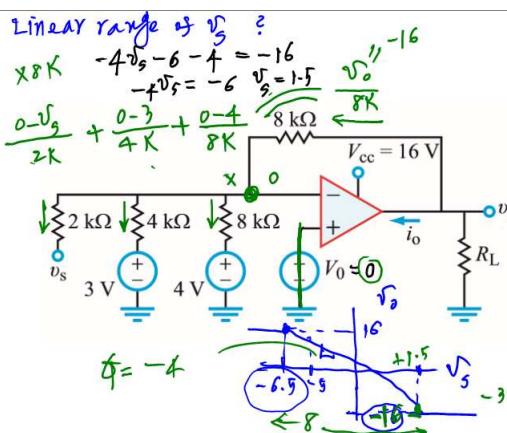
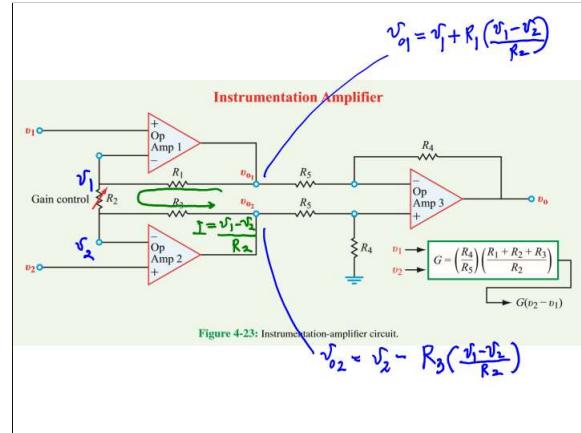
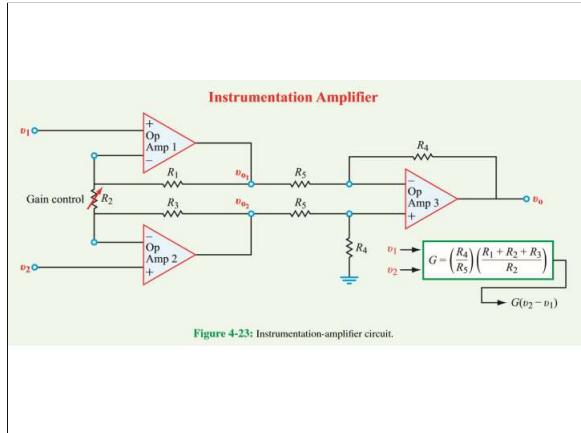


Figure P4.29: Circuit for Problems 4.29 through 4.31.





$$v_{o2} = v_2 - R_3 \left( \frac{v_1 - v_2}{R_2} \right) = v_2 + (v_2 - v_1) \frac{R_3}{R_2}$$

$$v_{o1} = v_1 + R_1 \left( \frac{v_1 - v_2}{R_2} \right) = v_1 + (v_2 - v_1) \left( -\frac{R_1}{R_2} \right)$$

$$v_{o2} - v_{o1} = (v_2 - v_1) + (v_2 - v_1) \left( \frac{R_3}{R_2} - \left( -\frac{R_1}{R_2} \right) \right)$$

$$= (v_2 - v_1) \left( 1 + \frac{R_3}{R_2} + \frac{R_1}{R_2} \right) = (v_2 - v_1) \frac{R_1 + R_2 + R_3}{R_2}$$

$G_1 = \frac{R_1 + R_2 + R_3}{R_1}$ 
  
 $v_o = G_2 (v_{o2} - v_{o1})$

$$v_{o2} - v_{o1} = (v_2 - v_1) \frac{R_1 + R_2 + R_3}{R_2}$$

**Instrumentation Amplifier**

$v_b = v_{o2} - \frac{R_4}{R_4 + R_5} = v_a$

$I = \frac{v_a - v_{o1}}{R_5}, v_o = R_4 I + v_a$

$v_o = R_4 \frac{v_a - v_{o1}}{R_5} + v_a$

Figure 4-35: Wheatstone-bridge op-amp circuit.

$$v_o = R_4 I + v_a$$

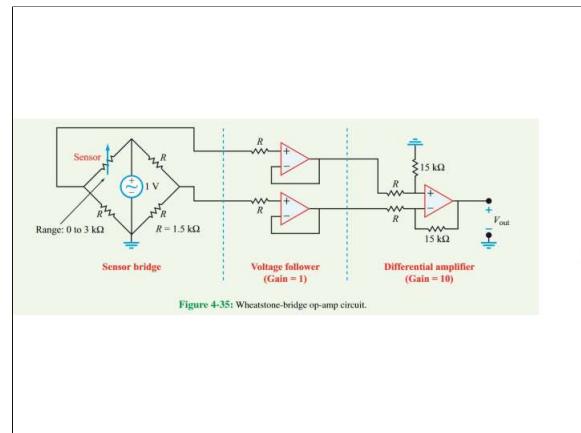
$$= R_4 \frac{v_a - v_{o1}}{R_5} + v_a$$

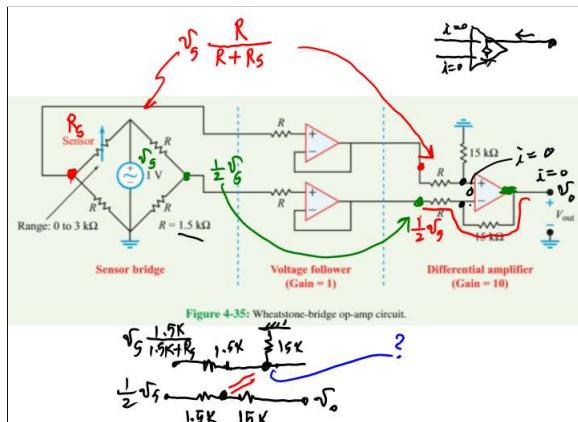
$$= v_a \left( \frac{R_4}{R_5} + 1 \right) - \frac{R_4}{R_5} v_{o1}, \text{ where } v_a = v_{o2} \frac{R_4}{R_4 + R_5}$$

$$= v_{o2} \frac{R_4}{R_4 + R_5} \left( \frac{R_4 + R_5}{R_5} \right) - \frac{R_4}{R_5} v_{o1}$$

$$= v_{o2} \frac{R_4}{R_5} - \frac{R_4}{R_5} v_{o1} = \frac{R_4}{R_5} (v_{o2} - v_{o1})$$

$$\Rightarrow v_o = \frac{R_4}{R_5} (v_{o2} - v_{o1}) = \left[ \frac{R_4}{R_5} \frac{R_1 + R_2 + R_3}{R_2} \right] (v_2 - v_1)$$





$$v_1 = \frac{R}{R+R_s} \times 1 = v_{o1}$$

$$v_o = (v_{o1} - v_{o2}) \frac{15k}{R}$$

$$4 \times 10 = 10 \Rightarrow R = 1.5k$$

Figure 4-35: Wheatstone-bridge op-amp circuit.

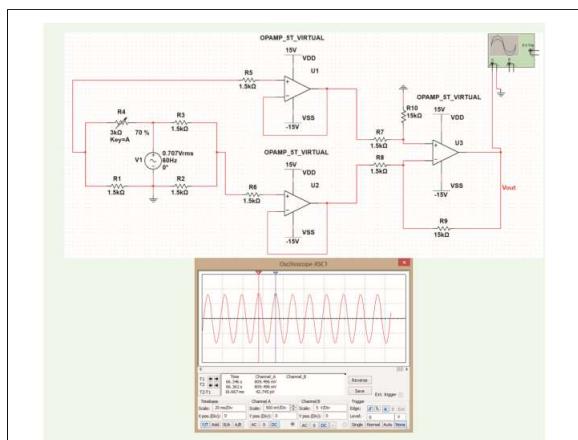
$$v_2 = \frac{R}{R+R_s} \times v_g = 0.5 v_g = v_{o2}$$

$$\text{For } R_s = R, v_o = 0$$

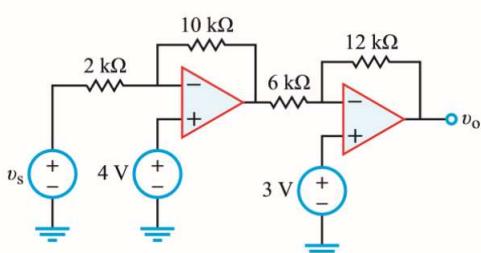
$$2R, v_o = \left(\frac{1}{2} - \frac{1}{2}\right) 10 v_g = -\frac{1.5k}{2} v_g$$

$$= \left(\frac{1.5k}{1.5k+R_s} - \frac{1}{2}\right) 10 v_g$$

$$= \left(\frac{1.5k}{3k} - \frac{1}{2}\right) 10 v_g$$



4.51 Solve for  $v_o$  in terms of  $v_s$  for the circuit in Fig. P4.51.



\*3.89 The two-transistor circuit in Fig. P3.89 is known as a **current mirror**. It is useful because the current  $I_0$  controls the current  $I_{REF}$  regardless of external connections to the circuit. In other words, this circuit behaves like a current-controlled current source. Assume both transistors are the same size such that  $I_{B1} = I_{B2}$ . Find the relationship between  $I_0$  and  $I_{REF}$ . (Hint: You do not need to know what is connected above or below the transistors. Nodal analysis will suffice.)

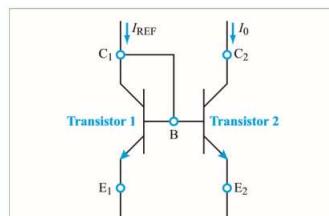


Figure P3.89: A simple current mirror (Problem 3.89).

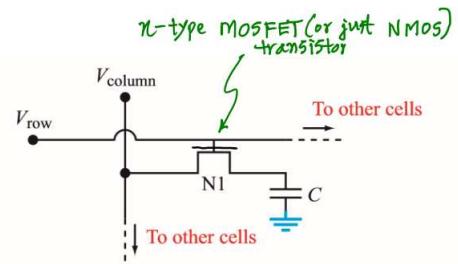
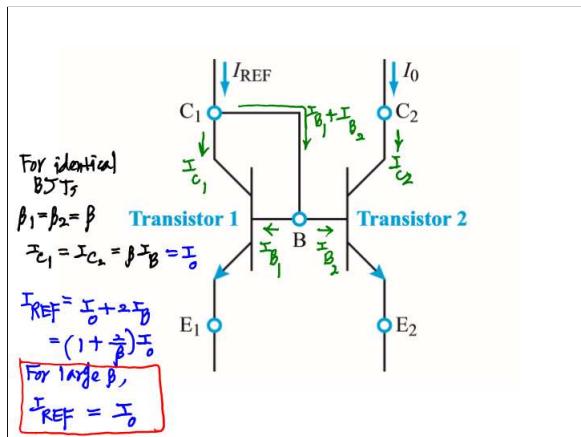
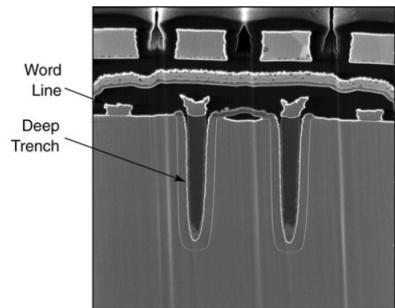
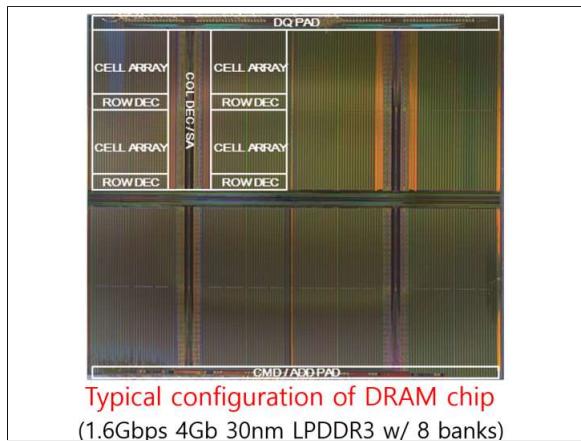
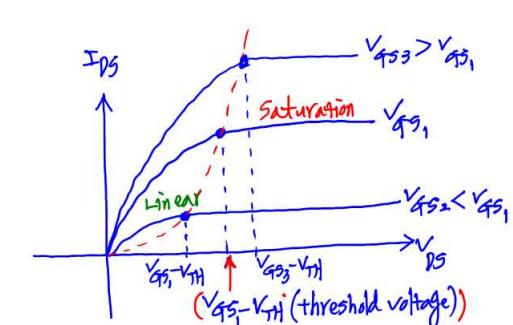
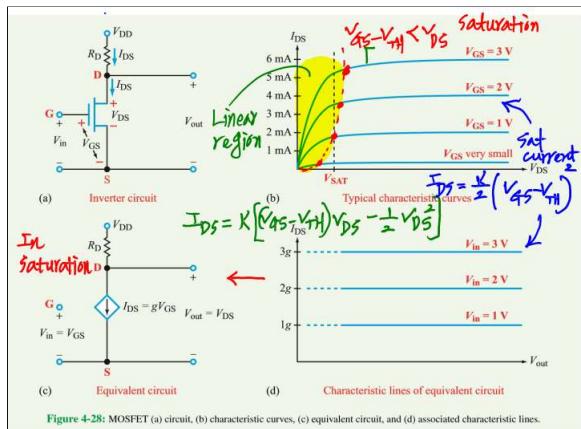


Figure TF10-3: 1-bit DRAM cell.



(b) DRAM cell with a trench cap.



4.61 In Problem 3.73 of Chapter 3, we analyzed a current mirror circuit containing BJTs. Current mirror circuits also can be designed using MOSFETs, as shown in Fig. P4.61. Determine the relationship between  $I_0$  and  $I_{\text{REF}}$ .

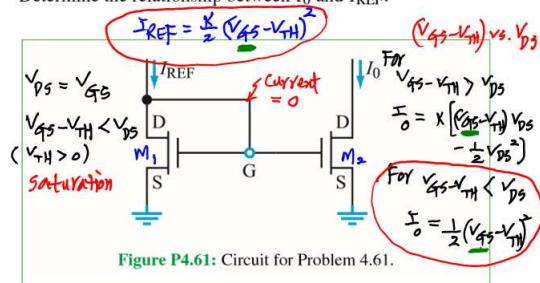


Figure P4.61: Circuit for Problem 4.61.

$$I_{\text{REF}} = I_0 \text{ when } M_2 \text{ is in saturation}$$

$$\begin{aligned} dq &= C dV \\ q &= C V \rightarrow \frac{dq}{dt} = \frac{d}{dt}(CV) \\ i &= C \frac{dV}{dt} \end{aligned}$$

C time invariant

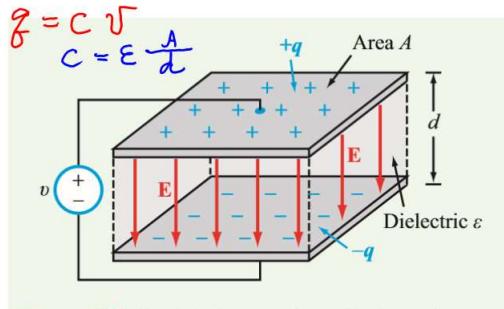


Figure 5-11: Parallel-plate capacitor with plates of area  $A$ , separated by a distance  $d$ , and filled with an insulating dielectric material of permittivity  $\epsilon$ .

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/cm}$$

Table 5-2: Relative electrical permittivity of common insulators:  $\epsilon_r = \epsilon/\epsilon_0$  and  $\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$ .

Material	Relative Permittivity $\epsilon_r$
Air (at sea level)	1.0006
Teflon	2.1
Polystyrene	2.6
Paper	2-4
Glass	4.5-10
Quartz	3.8-5
Bakelite	5
Mica	5.4-6
Porcelain	5.7
oxide	3.9

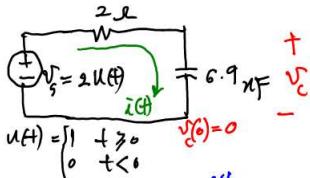
$$\begin{aligned} A &= 1 \mu\text{m} \times 2 \mu\text{m} = 2 \times 10^{-12} \text{ m}^2 \\ d &= 10 \text{ nm} = 10 \times 10^{-9} \text{ m} \\ C &= \epsilon \frac{A}{d} \\ &= 3.9 \times 8.854 \times 10^{-12} \text{ F/m} \times \frac{2 \times 10^{-12} \text{ m}^2}{10^{-8} \text{ m}} \\ &= 6.906 \times 10^{-15} \text{ F} = 6.906 \text{ pF} \end{aligned}$$

$$\begin{aligned} \int I(t) dt &= \int_0^{10} 1 \text{ mA} dt \quad t < 0 \\ C &= 6.906 \text{ pF} \\ q(t) &= \int_0^t I(\tau) d\tau = \int_0^t 1 \text{ mA} d\tau \\ &+ \int_0^t I(\tau) d\tau = 10 \text{ mA} \cdot t \\ V(t) &= \frac{q(t)}{C} = \frac{10 \text{ mA} \cdot t}{6.906 \text{ pF}} \end{aligned}$$

$t[\text{ns}]$

$$\begin{aligned} 0 & \quad V \\ 1 & \quad \frac{10 \times 10^{-6} \times 10^{-9}}{6.906 \times 10^{-15}} = 1.45 \text{ V} \\ 2 & \quad 2.9 \text{ V} \end{aligned}$$

problem = Find  $i(t)$



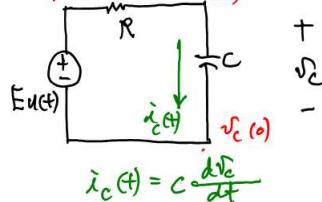
$$u(t) = \int_0^t i(\tau) d\tau$$

$$v_c = \frac{1}{C} \int_0^t i(\tau) d\tau = \frac{1}{6.9 \text{ mF}} \int_0^t i(\tau) d\tau$$

$$i(t) = \frac{v_s - v_c}{R}$$

$$KVL \quad v_s = 2i + \frac{1}{6.9 \text{ mF}} \int_0^t i(\tau) d\tau$$

prob = Find  $v_c(t)$



current will stop only when  $v_c = E$

$$i_c(t) = C \frac{dv_c}{dt}$$

$$KVL \quad E u(t) = R i_c(t) + v_c(t)$$

$$= R C \frac{dv_c}{dt} + v_c(t)$$

$$\text{For } t > 0 \quad [E = R C \frac{dv_c}{dt} + v_c]$$

In general

$$RC \frac{dv_c}{dt} + v_c = E$$

$$\frac{dv_c}{dt} + \frac{1}{RC} v_c = \frac{E}{RC} \quad a = \frac{1}{RC}$$

$$\boxed{\frac{dv_c}{dt} + a v_c = b}$$

multiply both sides by  $e^{at}$

$$e^{at} \frac{dv_c}{dt} + a e^{at} v_c = b e^{at}$$

Integrating both sides

$$\int e^{at} \frac{dv_c}{dt} dt + \int a e^{at} v_c dt = \int b e^{at} dt$$

$$\Rightarrow \int_0^t \frac{d}{dt}(v_c e^{at}) dt = \int_0^t b e^{at} dt$$

$$v_c e^{at} - v_c(0) = \int_0^t \left( \frac{b}{a} e^{at} \right) dt$$

$$v_c e^{at} - v_c(0) = \frac{b}{a} (e^{at} - 1)$$

$$v_c e^{at} = v_c(0) + \frac{b}{a} (e^{at} - 1)$$

$$v_c(t) = v_c(0) e^{-at} + \frac{b}{a} (1 - e^{-at})$$

$$= \frac{b}{a} + (v_c(0) - \frac{b}{a}) e^{-at}$$

$$= v_c(0) + (v_c(0) - \frac{b}{a}) e^{-at} \quad (5.96)$$

+ 1.5V battery

mAh (milli-ampere hour) measures battery capacity. In other words - how much current a battery will discharge over a one hour period. Higher mAh ratings correspond to how long a current can be drawn, rather than how fast it can be drawn. The mAh abbreviation is also written as Ah or Ampere-hour. (1 Ah = 1,000 mAh). Overall capacity is influenced by factors like temperature and speed of discharge. A 40 mAh battery can discharge 40 millamps for one hour, 20 millamps for two hours, and so on.

40mAh battery  $\rightarrow$  charge capacity

$$Q = I \times T = 40[\text{mA}] \times 3600[\text{s}] = 144[\text{A} \cdot \text{s}]$$

$$\boxed{\text{1.5V} \quad 7\Omega \quad 96\text{F}}$$

$$\frac{Q}{V} = C = \frac{144}{1.5} = 96\text{F}$$

$$v_c(t) = v_c(0) + (v_c(0) - v_c(\infty)) e^{-\frac{t}{RC}}$$

$$1.5V = 1.5 + (0 - 1.5) e^{-\frac{t}{(6 \times 96)}} \\ \text{initial voltage } 0 \quad = 1.5 \left( 1 - e^{-\frac{t}{576}} \right)$$

$$t = 1 \text{ hour} \quad e^{\frac{1}{576}} = 0.0047 \approx 0$$